# CONTRACT AND STRATEGIC NETWORK DESIGN

# FOR REVERSE PRODUCTION SYSTEMS

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Joshua W. Pas

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# CONTRACT AND STRATEGIC NETWORK DESIGN

# FOR REVERSE PRODUCTION SYSTEMS

Approved by:

Dr. Jane C. Ammons, Co-Advisor
School of Industrial and Systems Engineering
*Georgia Institute of Technology*

Dr. Matthew J. Realff, Co-Advisor
School of Chemical and Biomolecular Engineering
*Georgia Institute of Technology*

Dr. Paul M. Griffin
School of Industrial and Systems Engineering
*Georgia Institute of Technology*

Dr. Valerie Thomas
School of Industrial and Systems Engineering
*Georgia Institute of Technology*

Dr. Robert Peoples
Executive Director
*Carpet America Recovery Effort*

Date Approved: March 13, 2008

*To my parents, Kenneth and Jutharat*

*To my sister, Yotkhwan*

*To my wife, Darat*

*Thanks for your love and support through the years*

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# SUMMARY

A Reverse Production System (RPS) is a network of transportation logistics and processing functions that collects, refurbishes, and demanufactures for reuse/recycle used or end-of-life products. In this thesis, I focus on the RPS strategic decisions of a processor and collectors when the collection network and contracts for materials can be co-designed.

The research problem is motivated by the need of material processors to ensure a consistent flow of material from collectors at a cost that will enable them to be competitive with virgin raw materials. The failure to develop a cost-effective collection network can lead to poor overall economics where expensive processing assets are not fully utilized. The three key problems from the processor's point of view are: 1) how to design a strategic collection network; 2) how to be competitive in the collected materials market place when significant investment is at risk; and 3) how to avoid overpaying for materials when collectors are in regions with different volumes and costs.

The multiple goals of this research are: 1) to integrate the contract and strategic network design in RPS; 2) to develop contract mechanism designs to improve the performance under incomplete information and study the value of information (complete vs. incomplete); and 3) to introduce and analyze new strategic network models for effectiveness in solution quality and time.

Concepts of mathematical optimization, contract theory, and game theory are utilized in proposing models that couple contract and network problems, including lump sum and variable volume contracts.

# CHAPTER 1

# INTRODUCTION

In this chapter, I introduce the research topics of my thesis. First, I provide an overview of Reverse Production Systems in Section 1.1. Subsequently, I motivate the research in Section 1.2 and formally present the research problem in Section 1.3. Finally, I discuss the organization of the thesis in Section 1.4.

## 1.1 Reverse Production System Overview

Recycling end-of-life products is important today due to the increasing concern about both the environmental impact of discarded materials and the economic reuse value of such materials. A Reverse Production System (RPS) can be defined as a network of transportation logistics and processing functions that collect, refurbish, and demanufacture for reuse/recycle, used, or end-of-life products. An abstraction of forward and reverse production systems is shown in Figure 1.1, adapted from Realff et al. (1999). The products' recovery values and waste disposal avoidance costs are the heart of the economics of recycling. The recycler must balance these two factors with the costs associated with transportation, sorting, demanufacturing/reburbishing processes, and material recycling.

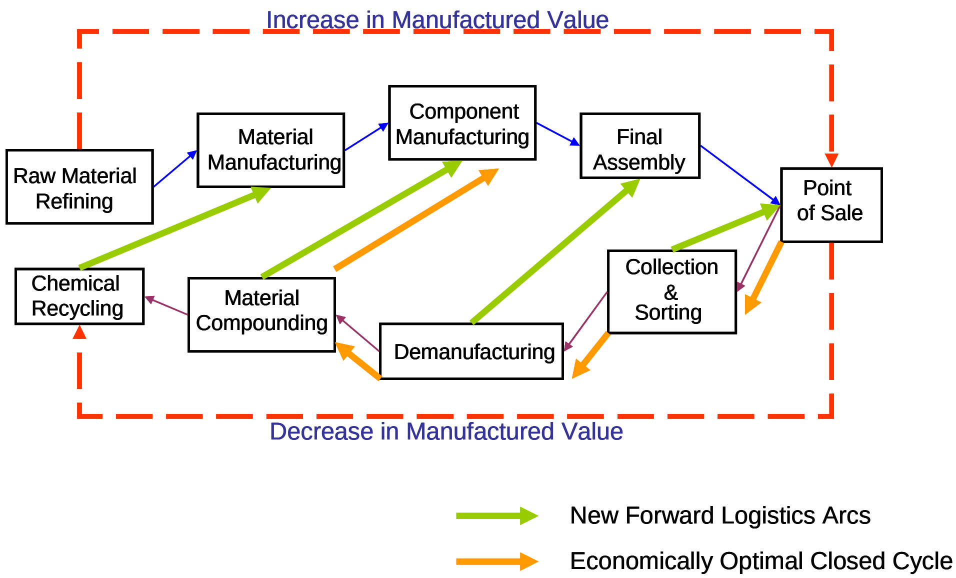



Figure 1.1: Abstraction of Forward and Reverse Production Systems (Realff et al. 1999)

While the financial and environmental benefits of handling return flows of supply chain production wastes, packaging, and end-of-life products are potentially substantial, the financial viability and efficiency of a RPS in today's marketplace is highly dependent on its infrastructure design and operational decisions. Decision tools can play an important role in strategically planning the design and growth of RPS infrastructure. A model of recycling system infrastructure is shown in Figure 1.2. There are four stages which include: supply (collected from sources of packaging, wastes, and end-of-life products), collection, processing, and demand (end users of refurbished products, components, and recycled materials), with transportation between stages. Strategic network design makes decisions on the locations of the collection and processing facilities, the type(s) of materials collected at collection facilities, the type(s) of processes installed at processing facilities, and the amount of materials collected, processed, and transported to various locations.

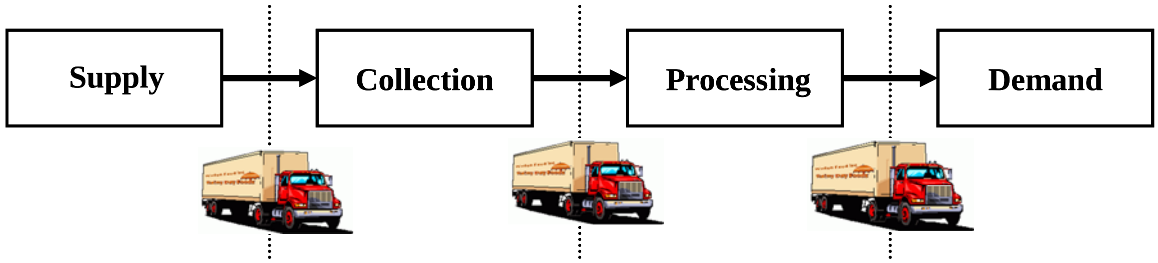



Figure 1.2: Recycling System Infrastructure

My research has been motivated by actual situations in the carpet industry. The major interest has been in integrating contract theory and strategic network design from the processor's point of view. Motivation details are given in the subsequent section.

## 1.2 Motivation

For certain industries, such as carpet and electronics, recycling infrastructure is beginning to mature with collectors and processors being established and significant volumes of material being collected and processed. As new technology for recycling is translated from research into practice, new avenues for the material to be recycled are being established. In addition, there is still a significant appetite for materials for export. In this situation, a new processor, or one who wishes to expand his or her existing network, must ensure that s/he can supply their processing facilities. These facilities may require significant capital investment to build. Moreover, a failure to secure a reliable supply of material could be fatal to the overall viability. Furthermore, the economics of recycling can be marginal, and if the processor does not successfully manage the cost of the supply of material, then this could render the operation economically inviable.

This situation presents the processor with the problem of making sure that the collection network is sufficiently incentivized to provide the material to the processor and not to sell it elsewhere, and yet do so for the lowest possible cost. In this thesis, I explore this question through the design of contracts that the processor should offer to the collectors and the concomitant design of the collection network.

One example of an industry that faces this problem is carpet recycling. The nylon fibers (used in carpet) produced in 1998, which should be disposed by approximately 2008, are worth a potential one billion U.S. dollars (Wongthatsanekorn 2006). Nevertheless, according to Carpet America Recovery Effort's Annual Report in 2006, 5 billion pounds of used carpet were discarded to landfill in 2006 while less than 5% was recycled. The renewal of Nylon 6 saves 4.4 Trillion BTUs of energy annually, enough power to heat over 100,000 U.S. homes each year, (International Fiber Journal 1999). The Carpet and Rug Institute (2003) has shown how the carpet industry has reduced its environmental footprint over the years. Not only was the magnitude of landfill use, energy consumption, $CO_2$ emissions, hazardous air pollutants, and water usage reduced by 80% per square yard over the past decade from 1990, but the production increased by 47% over the same period. The goal is to reduce the environmental footprint a further 28% by 2012. A significant component of this reduction could be achieved through more successful recycling systems for carpet.

The gross energy of virgin Nylon 6 is approximately 120.5 MJ/kg, (Boustead 2005). This includes the energy to produce Nylon 6 and the feedstock energy. The required transportation energies for heavy truck and rail are 985 J/kg-mile and 355 J/kg-mile (Center for Transportation Analysis 2007), respectively. In transporting 1 kg of

Nylon 6 carpet by truck for 500, 1,000, and 10,000 miles, the required transportation energies are 985 KJ, 2.0 MJ, and 19.7 MJ, respectively. For rail, the required transportation energies are 355 KJ, 710 KJ, and 7.1 MJ for distances of 500, 1,000, and 10,000, respectively. The upper bound on the energy consumption for Nylon 6 carpet recycling process can be obtained by the difference between the gross energy to produce virgin Nylon 6 and the transportation energy. If the processor is transporting carpet by truck, the upper bounds on energy consumption for recycling process are 119 MJ, 118 MJ, and 100.3 MJ for distances of 500, 1,000, and 10,000 miles, respectively. For rail, these upper bounds are 119.6 MJ, 119.3 MJ, and 112.9 MJ, respectively. The mechanical energy to recycle 1 kg of carpet will be significantly less than these upper bounds. This suggests that even when the processor needs to transport carpet for 10000 miles (via truck or rail), there is clearly energy benefit for recycling Nylon 6 carpet.

As explained in Wongthatsanekorn (2006), two major carpet recycling companies suffered financial problems that led to their shutdowns. Evergreen Nylon Recycling, located in Augusta, GA, was shut down in 2001, (Atlanta Business Chronicle 2001). Polyamid 2000, located in Premnitz, Germany, declared bankruptcy in 2003, (Carpet America Recovery Effort 2004). In both cases, a key contributing factor was the high cost of supplied material. The strategic planning of collection networks to supply the processing plant is deemed a critical factor in the success (or failure) of recycling operations. The solution must include good network design of collection facilities and associated contractual arrangements to ensure high quality and appropriate flows of material. This concept is the key motivation in developing the models and solution approaches in this thesis.

Using the above motivation, I formally describe the research problem in the next section.

## 1.3 Research Problem

The research problem statement can be written as:

*From the point of view of the processor when s/he has incomplete information about potential collectors, how should contract alternatives for potential collectors be designed and how should a network of collection facilities be strategically chosen to meet the target quantity and budget constraints, while aiming to maximize the processor's net profit?*

The processor's problem is considered when s/he does not have collectors' private information on type. This collector type is based on marginal cost. For more details on the definition and specification of types, see Section 5.1. I assume that there are many types of collectors with different marginal costs. Without this private information, the processor is uncertain about the performance of each collector. To provide incentives to collectors to align their objectives with the processor's own goals, a ***contract*** is designed. A contract is an agreement between two or more parties in which an offer is made and accepted.

In this thesis, two contract designs are proposed: lump sum and variable volume contracts. The lump sum contract utilizes the principal-agent concept, which is discussed in Chapter 3. The processor (principal) provides a simple menu contract, which includes many contract alternatives, to the set of potential collectors. This menu contract specifies collected quantity (lb) and lump sum transfer payment ($). Each collector selects the contract alternative that is most beneficial for his or her type of operation. From the revelation principle, see Section 3.1, collectors truthfully select the contract alternative designed for his or her type because it is also the most beneficial alternative. For the variable volume contract, different contract structures are offered to each collector. This contract specifies the dollar per pound, and then lets the collector determine the collected quantity (lb). The selected contract has embedded incentive for collector not to deviate from the promised quantity. It utilizes Nash Equilibrium concept, which is discussed in Chapter 7.

Concurrent to the design of contracts, the processor wants to establish an overall collection network. From the list of potential collectors, the processor must select a subset of collectors to provide enough total material (meet the target quantity) to the processing facility. Moreover, collectors are chosen in such a way that the total material cost is within the processor's budget. In designing this collection network, transportation costs and fixed costs (transport and administration) are incorporated into the processor's net profit. Figure 1.3 illustrates the problem.

I hypothesize that co-designing the collection network and contracts for materials at the same time should produce a higher processor's net profit than first designing the contract and network individually and later combining these decisions.

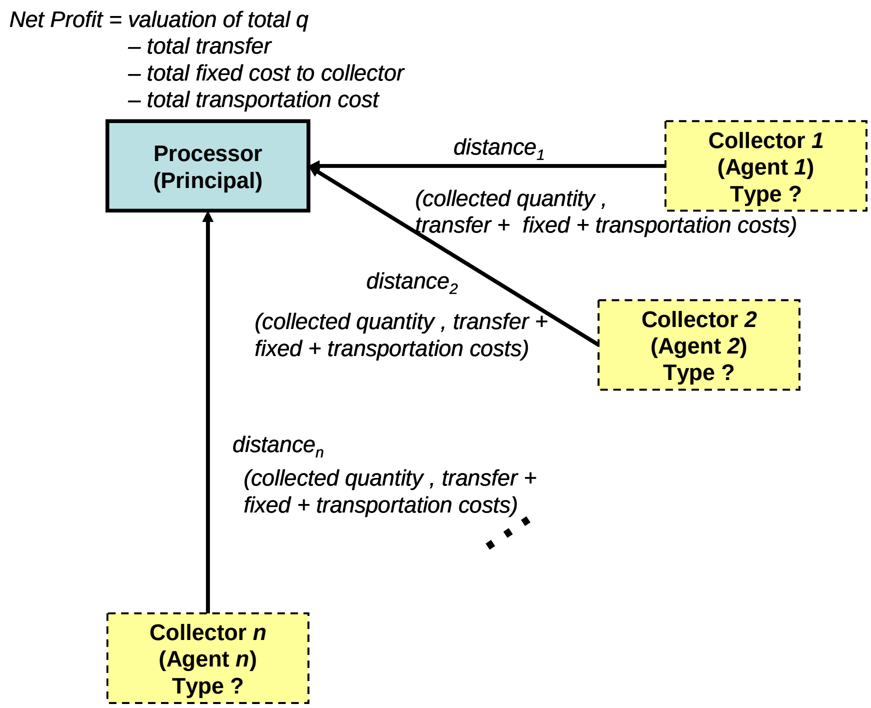



Figure 1.3: Contract & Strategic Network Problem Overview

The research goals and key assumptions are described in the following sections. These key assumptions are held in most of the models; if any of these assumptions is relaxed in addressing specific situations, it will be clearly stated.

### 1.3.1 Key Assumptions

1) One (recycle/reuse) product,

2) One processor, many collectors,

3) One can group collectors into different types, based on characteristics such as their efficiency,

4) Each collector collects from his or her region, and the presence of other collectors does not influence the collector, and

5) Each collector's collected quantity to processor's overall target quantity ratio (scaling parameter) has relatively small value.

Assumptions 1 and 2 define the scope of the research analysis. Assumption 3 initiates the analysis with two collector types. Assumption 4 enforces no cross-region collection and no impact of competition between collectors. Finally, Assumption 5 states that a processor needs to select a significant number of collectors in order to achieve his or her overall target quantity. If there are only a few chosen collectors, the processor has a potential problem of defection so that for any unforeseeable reason prohibiting a collector's operations, the processor's network could entirely collapse. Again, assumptions can be relaxed as different problems are addressed.

### 1.3.2 Goals

My research goals are: 1) to integrate the contract and strategic network design in RPS planning; 2) to study the value of information (complete vs. incomplete); 3) to develop contract mechanism designs to improve system performance under incomplete information; and 4) to introduce and analyze new strategic network planning models for effectiveness in solution quality and time. I apply the models for a recycled carpet application to evaluate my models.

With the above description of research problem, key assumptions, and research goals, the thesis chapters are outlined in the next section.

## 1.4 Organization of the Thesis

My thesis is organized as follows. The relevant literature review is discussed in Chapter 2. Chapter 3 provides background and proposed models for the contract problem. The different models developed in the thesis are presented in Figure 1.4. The leaves of the tree are the six different models that are developed and solved. The tree has two main branches, one for the contract and network problem and one for the network problem on its own.

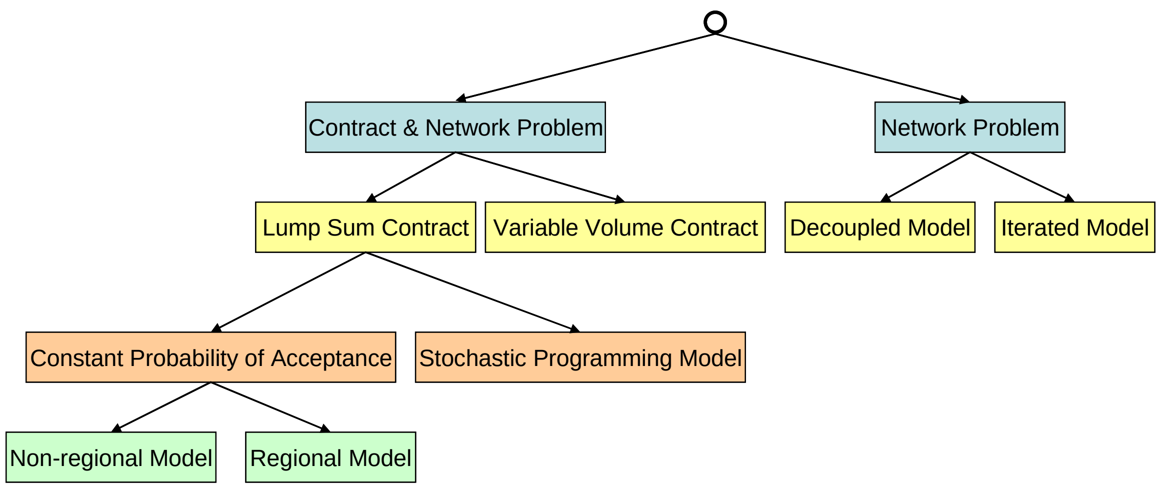



Figure 1.4: Research Tree

In Chapter 4, I present three alternative approaches to determine the RPS strategic network decisions, which are the right branch of the tree: Coupled, Decoupled, and Iterated Models. I denote a *coupled system* as a comprehensive representation that simultaneously considers every component in the RPS infrastructure to determine both *collection decisions* and *processing decisions*. This model has been presented previously in research done within the RPS group at Georgia Tech (Newton (2000) and

Assavapokee (2004)). With a *decoupled system*, I first examine only the supply and collection stages to determine the collection decisions, and for the second step determine the processing structure based on the collection decisions. The *iterative* strategy employs clustering techniques for situations where the first two approaches do not perform well. The effectiveness of each approach is measured by solution quality and computational time, using the more traditional Coupled Model as the baseline.

In Chapters 5, 6, and 7, I focus only on the system of processor and collectors that operate between the supply and demand stages. The aim is to study two different types of contracts for establishing a collection network of used materials, namely a lump sum contract and a variable volume contract. A lump sum contract specifies the quantity and monetary decision pair. On the other hand, a variable volume contract specifies the contract structure and quantity. Using optimization and stochastic programming as tools, concepts of game theory and contract (incentive) theory are utilized in the study. A major part of the research utilizes the concept of an incomplete information game.

The proposed contract and network lump sum models are presented in Chapter 5 which include sequential and simultaneous models of non-regional and regional problems. A Stochastic Programming approach to co-design the lump sum contract and strategic collection network is presented in Chapter 6. Chapter 7 presents the proposed variable volume models, which are quite different from the lump sum contract. Finally, the summary, contributions, and future directions are discussed in Chapter 8.

# CHAPTER 2

# BACKGROUND & LITERATURE REVIEW

In this chapter, I present the research background and a review of the published literature related to various aspects of the research. This thesis draws upon a wide range of modeling and algorithmic techniques. The review of each area therefore focuses on the information that is most pertinent to the development of the thesis. Section 2.1 reviews the literature in the area of reverse logistics network design. A brief review of (forward) supply chain management is presented in Section 2.2. Section 2.3 discusses fundamental literature in the area of mixed integer linear programming, whereas Section 2.4 reviews the literature in the contract theory field. A review is presented in Section 2.5 which stresses the literature in dynamic and complete information games. Section 2.6 reviews literature in stochastic programming optimization. Lastly, literature of the mixed integer nonlinear programming area is reviewed in Section 2.7.

## 2.1 Reverse Production Systems

The economic importance of Reverse Production Systems (RPS) has recently become much better understood. According to Rogers and Tibben-Lembke (1999), reverse logistics cost approximately US $35 billion in 1997. Fleischmann et al. (2004) show that both the environmental consciousness of consumers and environmental laws have stimulated RPS development.

A broad overview of recycling logistics is given by Flapper (1995), Flapper (1996), and Fleischmann et al. (2000). A survey on environmentally-conscious manufacturing and product recovery can be found in Gungor and Gupta (1999). Also using a coupled approach, Spengler et al. (1997) propose a location-allocation model in determining the infrastructure of reclamation facilities and demanufacturing plants. Srivastava (2008) replicates published optimization concepts in the reverse logistics network design in his case study. Fleischmann et al. (2004) distinguish the reverse logistics facility location models according to the network scope and the supply type (push/pull).

Strategic planning, design, and implementation of the reverse logistics can be found in Realff et al. (2000), Dowlatshahi (2005), Pochampally and Gupta (2005), Walther et al. (2008), and Jacobs and Subramanian (2008). Given that uncertainty is a key characteristic of product recovery networks, Hong et al. (2006), Inderfurth (2005), Listes and Dekker (2005), and Realff et al. (2004) specifically address this issue. Other reverse logistics network models include Barros et al. (1998), Marin and Pelegrin (1998), Schultmann (2006), and Thierry (1997). RPS infrastructure design has been seen as a large-scale optimization problem in Spengler et al. (1997) and Realff et al. (2004) while it has been approached through the use of genetic algorithms in Min et al. (2006) and Zhou et al. (2005).

## 2.2 (Forward) Supply Chain Management

Although I center attention on the reverse supply chain, the variable volume contract models can be generalized to the forward supply chain (see Section 7.6). My research specifically focuses on the strategy or design phase, addressing facility, transportation, and information decisions. Hence I present a brief literature review in (forward) supply chains, which has received much interest both practically and academically in recent years.

According to Chopra and Meindl (2003), a “supply chain consists of all parties involved, directly or indirectly, in fulfilling a customer request. The supply chain not only includes the manufacturer and supplies, but also transporters, warehouses, retailers, and customers themselves. ” They categorize decision phases in a supply chain into three groups, namely 1) supply chain strategy or design, 2) supply chain planning, and 3) supply chain operations. Decisions made in each phase have different time frames: years in the strategy phase; a quarter to a year in the planning phase; and weeks or days in the operations phase. Drivers of any supply chain performance include facilities, inventory, transportation, and information. Typical measurements of supply chain performance include profit, cost, quality, visibility, and responsiveness. Additional introductions to supply chain management can be found in Chopra and Meindl (2003), Simchi-Levi et al. (2003), Shapiro (2001), and Stadtler (2005).

Modeling of supply chains includes descriptive and normative (optimization) models, Shapiro (2001). The descriptive model is developed to understand functional relationships, including forecasting, costing, resource utilization, and simulation. On the other hand, the optimization model is constructed to help make good decisions. Both model types are widely studied in different application areas. Some of the most commonly studied fields include network configuration, supply chain coordination, decision-support systems, inventory control, vehicle routing and transportation, warehousing, information technology implementation, value of information, strategic alliances, and distribution strategies. Tayur et al. (1998) and Klose et al. (2002) present fundamental and advanced quantitative models for many of the above fields.

## 2.3 Mixed Integer Linear Programming Modeling

As a decision tool, mixed integer linear programming (MILP) is attractive due to its modeling capabilities and the availability of powerful commercial solvers. Using MILP for large RPS infrastructure design problems requires challenging solution requirements. MILP has motivated the employment of decomposition and relaxation methodologies for the Decoupled and Iterated formulations in my research. General concepts of optimization for large scale systems are presented in Lasdon (1970) and Wismer (1971), while Geoffrion (1971) has shown an important feature of large-scale mathematical programming is that most problems possess distinctive structures that can be exploited.

In my new Decoupled and Iterative approaches for RPS design presented in Chapter 4, the classic idea of problem decomposition has been used. Although my decomposition and relaxation methods are new for this application, they are based on well-established ideas. Classic decomposition techniques include Benders decomposition, Benders (1962), and Dantzig-Wolfe decomposition, Dantzig and Wolfe (1960). A summary of these two techniques and their generalizations, variable decomposition and constraint decomposition approaches, can be found in Olaf et al. (1993).

This thesis does not emphasize the development of new MILP algorithms. Rather, these algorithms are exercised as the solution tool for the contract and network problems.

## 2.4 Contract Theory

In many cases, information is held asymmetrically by market participants. Two examples include: 1) a firm hiring a worker and usually knowing less than the worker about the worker's natural ability, and 2) an auto insurance company insuring a driver and knowing less than the insured about his driving skill and the probability of his having an accident.

As discussed in Laffont and Martimort (2002), the delegation of a task to a worker who has different objectives than those of the firm is problematic when information about the worker is imperfect. This is at the core of worker incentive questions. If the worker has a different objective function but no private information, the firm can propose a contract that perfectly controls the worker without any delegation.

Conflicting objectives and asymmetric information are the two basic elements of contract (incentive) theory. Private information can be of two types: either the worker can take an action unobserved by the firm (*moral hazard, hidden action*), or the worker has private knowledge about his cost or valuation that is ignored by the firm (*adverse selection, hidden knowledge/type*). The allocation of resources is no longer ruled by the price system alone but also by contracts between asymmetrically informed partners. Because contract theory has changed the view of the function of organizations and markets, it has become an important area in microeconomic modeling.

Comprehensive discussions of contracts in supply chain management (SCM) are given by Tsay et al. (1999) and Cachon (2002). The former paper summarizes model-based research on contracts in the supply chain setting and provides a taxonomy for work in this area. The fundamentals of contract analysis, including supply chain structure and purposes of contracts, are addressed. The authors classify the literature in the contracting areas into specification of decision rights, pricing, minimum purchase commitments, quantity flexibility, buyback or returns policy, allocation rules, lead time, and quality.

In the latter paper, Cachon (2002) studies many supply chain models with various complexities. Optimal actions for each model are identified and incentives are created to obtain these optimal actions. A number of different contract types are identified with their benefits and drawbacks illustrated. The models are based on the newsvendor model, Silver et al. (1998) and Nahmias (1993). Although not complex, the newsvendor model is sufficiently rich to study three important questions in supply chain coordination. These questions are: 1) which contracts coordinate the supply chain, 2) which contracts have sufficient flexibility, and 3) which contracts are worth adopting. Other papers in SCM contracts include Anupindi and Bassok (1998), Bassok and Anupindi (1997), Cachon (1998), Drake and Swann (2005), Katz (1989), Lariviere (1998), Monahan (1984), Tsay and Lovejoy (1999), and Whang (1995).

As an overview of the principal-agent model, Laffont and Martimort (2002) provide a model where the principal delegates an action to a single agent through the take-it-or-leave-it offer of a contract. They make two implicit assumptions: 1) the bargaining issues are put aside and 2) a court of law is available to enforce the contract and impose penalties if one of the contractual partners deviates from the specification of the contract. The three types of information problems considered are adverse selection, moral hazard, and nonverifiability.

Another important summary of principal-agent theory is presented by Bolton and Dewatripont (2005). The authors present the basic ideas in incentive and information theory such as screening, signaling, and moral hazard. They give a comprehensive discussion of bilateral contracting, multilateral contracting, private information or hidden actions, auction theory, long-term contract, incomplete contracts, theory of ownership and control, and contracting with externalities. Again, a well-functioning legal system in the market is assumed.

The bilateral principal-agent model provides a flexible framework for studying a variety of important economic phenomena, ranging from planning problems such as the design of optimal social insurance programs, Diamond and Mirrless (1978), to contractual relations, Stiglitz (1974). Other more specific models in principal-agent include: 1) Bernheim and Whinston (1986) that address the situations where risk-neutral principals simultaneously and independently attempt to influence a common agent; 2) Segal (1999) that studies contracting between one principal and many agents in the presence of externalities and discusses inefficiencies from publicly and privately observed contracts; 3) Prat and Rustichini (2003) that introduce a complete information game with multiple principals and multiple common agents and applications such as lobbying (many principals, one agent) and multiple auctions. The decision of each agent affects the payoffs of all principals with each principal offering transfers to each agent, conditional on the agent's action. The Prat and Rustichini paper provides a general necessary and sufficient condition for the existence of pure-strategy equilibrium; and 4)

Spier (1992) that presents asymmetric information leading to contractual incompleteness, considering two types of costs, ex ante cost and ex post cost.

Although, there are several papers which employ games in network formation in social networks, markets, and electrical engineering, it is difficult to find contract models in network formation. The only known paper is Johari et al. (2006) which consider a network game where the nodes of the network wish to form a graph to route traffic. This is an application in the electrical engineering field which uses the idea of link stability equilibrium concept.

To my knowledge, no publication incorporates formal modeling of contracts in the field of reverse logistics. As a result, exploring the setting of contracts in reverse production systems is a major contribution of my doctoral research.

## 2.5 Game Theory (Dynamic and Complete Information)

Game theory is the study of multi-person decision problems, Gibbons (1992). The game employs strategic behavior, e.g., each person acts due to self interest. The basic elements of the game include player, type, belief, timing, action, and payoff with a few ways to categorize the games. A game can be cooperative or non-cooperative, simultaneous or sequential, involve perfect information or imperfect information, complete information or incomplete information. In a cooperative game, the players are able to form binding commitments, but cannot do so in a non-cooperative (competitive) game. As the name suggests, the simultaneous (static) game has all players moving or acting at the same time. In the sequential (dynamic) game, the player's move may precede one another. In the dynamic game, a player has perfect information if he knows the full history of the plays in the game up to his move. If not, it is an imperfect information game. The concept of complete information is different than the concept of perfect information. A player has complete information when all players' payoff functions are common knowledge to everyone.

The most widely used solution concept in applications of game theory is the Nash Equilibrium (NE) solution, Mas-Collell et al. (1995). In the NE, each player's strategy is the best response to the strategies actually played by others. Different notions of equilibrium exist for different classes of games. These are NE for static games with complete information, subgame-perfect NE (SPNE) for dynamic games with complete information, Bayesian NE for static games with incomplete information, and perfect Bayesian NE for dynamic games with incomplete information.

I consider a dynamic game with complete and perfect information in the variable volume contract analysis in Chapter 7. The complete information game involves players with common knowledge about each player's payoff function. In the perfect information game, at each move a player knows the full history of the plays up to that point. The goal of the game is to find the NE for each subgame and history. In the NE, no player would find it beneficial to deviate from a strategy, provided that all other players do not deviate from their strategies played at the Nash extreme.

A good introduction to game theory can be found in Gibbons (1992) and Varian (1992). Additionally, Fudenberg and Tirole (1991) present a more in-depth study of this topic.

A Stackelberg or Stackelberg leadership game (Stackelbergh 1934) is an economic game in which the leader moves before the follower (i.e., a complete and perfect information game). A corresponding model is solved by backward induction. Optimization is applied to the best response function in order to determine the SPNE solutions. Many real situations can be represented by a Stackelberg game because typical games are sequential with the more powerful player normally having the privilege of acting first. Numerous papers model the supply chain, mostly with two tiers, using a Stackelberg game. Aviv and Pazgal (2005) study the problem of finding optimal pricing for fashion-like seasonal goods, considering the seller as the leader and customers as followers. Baysar et al. (2007) research the effects of customer rebates and retailer incentives for the automotive industry. In their work, the manufacturer is the leader and the retailer is the follower. Pekgun et al. (2006) model the coordination of marketing and production for price and lead-time decisions, where the marketing and production departments are considered to be leader or follower in their settings.

Ferguson and Toktay (2005), Majumder and Groenevelt (2001), Savaskan et al. (2004), and Savaskan and Van Wassenhove (2006) apply game theory in their research on remanufacturing or reverse logistics field. Ferguson and Toktay (2005) develop models to support a manufacturer's recovery strategy with competition in remanufactured product markets. Majumder and Groenevelt (2001) present a two-period model with competition between the original equipment manufacturer (OEM) and a local manufacturer. Once the items are returned in the first period, the OEM and local manufacturer compete by setting prices, as depicted in Figure 2.1a). The demand function is utilized and complete information is assumed. Savaskan et al. (2004) consider a closed-loop supply chain with decentralized and centralized channels and with the manufacturer as the Stackelberg leader. They consider three decentralized models, which have returned products to 1) retailer, 2) manufacturer, and 3) third party collector, as in Figure 2.1b). Utilizing a market demand function, their decisions are wholesale price, product return rate, transfer price, and retail price. Lastly, the interaction between a manufacturer's reverse channel collection and forward channel pricing is studied in Savaskan and Van Wassenhove (2006). Examining direct and indirect collection systems, their decentralized channels employ a Stackelberg model, shown in Figure 2.1c). The manufacturer is still the Stackelberg leader with two retailers as followers. Similar decisions to Savaskan et al. (2004) are determined with additional competition analysis between retailers.

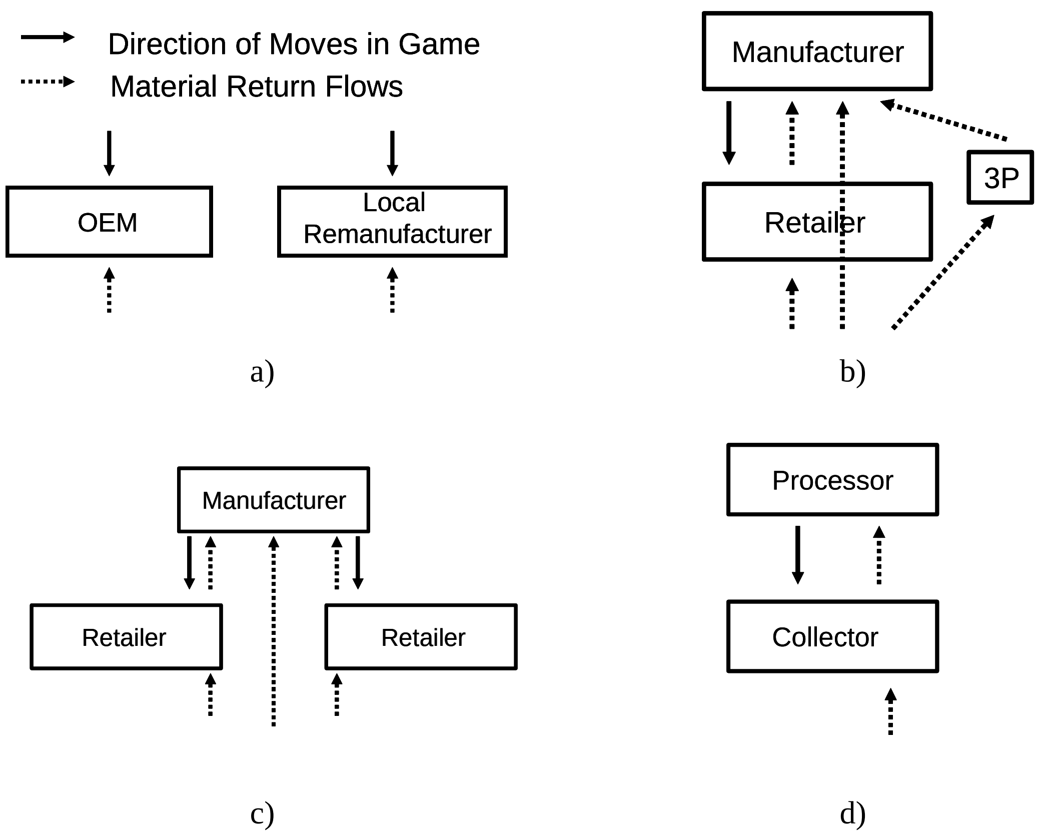



Figure 2.1: Examples of Reverse Logistics Games

To my knowledge, no one has studied the used product collector and processor contract structure without considering a market. Although Majumder and Groenevelt (2001) and Savaskan et al. (2004) implement a two-period game with complete information in remanufacturing, they consider competition between players. In contrast, I am looking at the coordination between a processor and a collector, with no competition. Because the reuse/recycle materials do not have a well established market and the flow of reuse/recycle materials is a small fraction of the overall market, it will fully use whatever the price. Although Savaskan and Van Wassenhove (2006) use market demand in setting price by maximizing net profit, with no competition, they have different decisions so that the present research is the only one to address contract structure using the Stackelberg game approach in a reverse logistics setting, shown in Figure 2.1d).

## 2.6 Stochastic Programming

An optimization model seeks decision variables that maximize or minimize objective functions while subjected to constraints limiting possible decision choices, Rardin (1997). Such models are deterministic if all parameter values are assumed to be certain. On the other hand, it is stochastic if there is uncertainty. Stochastic programming is viewed as mathematical programming with parameters that are random variables with specified distributions.

Typically, the impact of uncertainty is ignored or dealt with by sensitivity analysis or careful determination of instances parameters. When one has a large problem or a complex set of decisions, sensitivity analysis becomes inadequate. Moreover, one cannot eliminate inherent randomness no matter how careful he or she is, Linderoth (2007). Scenario analysis constructs or samples possible futures and then solves the corresponding problem for these values. Once one obtains a number of possible decisions, s/he either picks the best decision or combinations of decisions. Options have no value in scenario analysis and therefore never show up in the solution. It is often misunderstood that with enough scenarios, one will eventually obtain a correct solution. In hindsight, a scenario solution always turn out to be the best choice for that scenario. The setback is the same scenario solution is typically hardly optimal in all cases. In fact, most solutions are bad in all other cases, except for the scenario where they are best. On the contrary, even though the stochastic programming solution is never optimal after the fact, it is often a good solution for all cases, Kall and Wallace (1994).

Two-stage stochastic programming is the most widely applied and studied. Here one takes some actions in the first stage, after which a random event occurs affecting the outcome of the first-stage decision. The optimal policy is a single first-stage policy and a collection of recourse decisions identifying which second-stage action should be taken in response to each random outcome, Shapiro and Philpott (2007).

A summary of stochastic programming can be found in Kleywegt and Shapiro (2000), Birge and Louveaux (1997), Kall and Wallace (1994), and Shapiro and Philpott (2007). Stochastic programming has become an accepted approach to numerous applications, such as communication network (Liu 2005), crew scheduling (Yen and Birge 2006), financial planning (Carino et al. 1994), pension insurance (Hilli et al. 2007), product recovery (Listes and Dekker 2005), and supply chain management (Fisher et al. 1997).

In Chapter 6, the contract and strategic network problem can be modeled as a two-stage stochastic programming problem. In the first stage, the processor determines 1) regions to operate, 2) contracts for each region, and 3) collection hub(s) for each region. The uncertainty is whether each collection site accepts or rejects the given contracts. In the second stage, the processor determines the assignment of collections to hubs and the vehicle routing.

## 2.7 Mixed Integer Nonlinear Programming (MINLP) Modeling

Nonlinear programming (NLP) relaxes the key assumption of a linear objective function and constraints of the linear programming. Even though the linearity assumption holds for many practical problems, it often does not hold. Countless physical processes and properties are not linear. An introduction to NLP can be found in Hillier and Lieberman (2001) and Winston (2004). Both provide basic examples, properties, formulations, examples, and solving methods. More in-depth references can be found in Bazaraa et al. (2006), Bertsekas (1999), and Nash and Sofer (1996). One can find various solution methods and proofs in all these books.

The binary variables are very useful in modeling yes/no decisions, enforcing logical conditions, enforcing disjunctions, modeling fixed costs, and modeling piecewise linear functions. When a variable associates with a physical indivisible entity, one must consider an integer variable. Both the binary and integer variables are the mixed integer (MI) term in the mixed integer nonlinear programming (MINLP). There are numerous applications of the MINLP such as chemical engineering (Kocis and Grossmann 1988), gas transmission (De Wolf and Smeers 2000), and scheduling problems (Jain and Grossmann 1998).

Classical solution methods for MINLP include classical Branch-and-Bound, Outer Approximation (Duran and Grossmann 1986), Benders Decomposition, and LP/NLP-based Branch-and-Bound (Akrotirianakis et al. 2001). Modern developments in MINLP are summarized in Leyffer and Linderroth (2005).

I implement MINLP for my models in Chapters 5 and 6. With the binary and continuous decisions with nonlinear objective function and constraints, the MINLP allows more accurate representations of the underlying problem. Again, I would like to stress that this thesis does not contribute to MINLP algorithm development, but applies MINLP as a modeling tool for the contract and network problems.

The above sections review the related literature to this research problem. As RPS continues to be an important research area, the exploration of the strategic network and contract design is targeted. Problem definitions and proposed models are discussed next in Chapter 3.

# CHAPTER 3

# CONTRACT THEORY BACKGROUND & MODELS

Contract theory background and models are presented in this chapter. Section 3.1 discusses the Classical Principal-Agent (CPA) model for both complete and incomplete information contract models. Multiple Type Principal-Agent (MTPA) models are discussed in detail in Section 3.2, again with both complete and incomplete information contract models provided.

The CPA model is the fundamental contract model in this research. This model shows how a contract for different player types can be designed, while ensuring participation as well as simultaneously giving incentives. The MTPA model is a generalization of the CPA model. While the research focus is on the incomplete information contract, a complete information case is also discussed for all proposed models. In understanding the value of information, it is crucial to determine the best the processor can do in terms of net profit when all the information is available.

## 3.1 Classical Principal-Agent (CPA) Model

The Classical Principal-Agent (CPA) Model and notation are adapted from Laffont and Martimort (2002). A contract is an agreement between two or more parties in which an offer is made and accepted. Parties in a contract may not have all information about each other's abilities or types. Moreover, information usually is only revealed if it is in the agent's best interest. The CPA game is a one-shot game. The principal (processor) wants the agent (collector) to supply $q$ units of (recycle/reuse) resource. The principal's valuation for the $q$ units of the resource is *S(q)*. Assuming that $S'(q) > 0,\ S''(q) < 0,$ and $S(0) = 0$, the marginal value of goods is positive and strictly decreasing.

In one version of the model, the agent receives a lump sum transfer of funds from the principal when accepting the contract. In the simplest case, two types of agents are assumed: an agent is either efficient or inefficient depending upon the agent's marginal cost, $\theta$. The marginal cost for an efficient agent is denoted by $\underline{\theta}$, and by $\overline{\theta}$ for an inefficient agent, where $\overline{\theta} - \underline{\theta} > 0$. Note that the pairs (principal, agent) and (processor, collector) are interchangeable.

The objective of the principal is to propose a menu of contracts appropriate for both types of agents. The menu contract, which specifies collected quantity and lump sum transfer payment, has to be designed to enforce the behavior so that each agent will truthfully pick the right contract for his or her type. This truthful selection is a result of the Revelation Principle, Gibbard (1973) and Myerson (1979), which is presented in Section 3.1.2.

The game summary can be described as follows:

**Players**: Agent (collector) and Principal (processor)

**Types**: Agent – efficient or inefficient; Principal – none

**Beliefs**: Agent – none, Principal - prob(efficient) = *p*, prob(inefficient) = *1-p*

**Timing & Actions**: Dynamic with the following moves:

1) Nature determines the type of collector

2) The principal offers a contract – as functions of wage and effort (in this case material quantity is used to represent the effort).

3) The agent chooses the (wage, quantity) decision pair to maximize his or her utility.

Next the notation, terms, and decision variables are defined in order to develop the contract models used throughout this research.

Notation

$q$: Amount material (lb) provided by an agent (collector), can take values of $\underline{q}$ or $\bar{q}$

$t$: Transfer payment (lump sum) received by the collector for $q$ pounds of material, can take values of $\underline{t}$ or $\bar{t}$

$\underline{\theta}$: Marginal cost for an efficient collector

$\bar{\theta}$: Marginal cost for an inefficient collector, $\bar{\theta} - \underline{\theta} > 0$

$F$: Fixed cost of a collector, both types

$C(q, \underline{\theta}) = \underline{\theta} q + F$: Cost function of efficient collector

$C(q, \bar{\theta}) = \bar{\theta} q + F$: Cost function of inefficient collector

A : Set of feasible allocations $\mathrm{A} = \{(q,t) : q \in \Re^{+}, t \in \Re\}$

$S(q)$ : Valuation of the $q$ units to the principal, $S'(q) > 0,\ S''(q) < 0,$ and $S(0) = 0$

Terms

Principal's net profit ($): $S(q) - t$

Agent's utility ($): $t - \theta q$

Information rent ($): $t - \theta q$, which is generated by the information advantage of the agent over the principal

Decision Variables

$\underline{t}$ : Lump sum transfer payment to an efficient collector, \$

$\bar{t}$ : Lump sum transfer payment to an inefficient collector, \$

$\underline{q}$ : Quantity provided by an efficient collector, lb

$\bar{q}$ : Quantity provided by an inefficient collector, lb

Using this notation, first I present the CPA model with complete information in the next section.

### 3.1.1 Complete Information

In the CPA problem with complete information, the processor knows what type the collector is and is able to offer the correct contract to maximize the principal's net profit. This net profit consists of a nonnegative valuation and transfer payment to the collector. The only constraint ensures that the collector has an incentive to participate and in general is termed the participation constraint (*PC*). The processor's contracts for efficient and inefficient agents can be solved using the following models.

**CPA Model - Efficient Contract**

$$\begin{aligned} \text{Maximize} \quad & S(\underline{q}) - \underline{t} \\ \text{s.t.} \quad & \underline{t} - \underline{\theta}\underline{q} \geq 0 \\ & \underline{t}, \underline{q} \geq 0 \end{aligned}$$

**CPA Model - Inefficient Contract**

$$\begin{aligned} \text{Maximize} \quad & S(\bar{q}) - \bar{t} \\ \text{s.t.} \quad & \bar{t} - \bar{\theta}\bar{q} \geq 0 \\ & \bar{t}, \bar{q} \geq 0 \end{aligned}$$

It is unlikely that the processor has complete information for each collector type. However in hindsight, the processor can always compute his or her net profit for each collector type. In obtaining these additional solutions with the complete information, knowledge about the value of information is gained.

Having discussed the complete information variation, the incomplete information case is presented in the next section.

### 3.1.2 Incomplete Information

Incomplete information is a situation where each player knows his or her own payoff function, but may be uncertain about the other player's payoff functions. In this research, incomplete information concerns the type of the collectors, which is an adverse selection problem. Although the parametric form of the payoff is known, the specific value of the parameter is unknown. In this case, the processor does not know the agent's private information (type), but the probability distribution of this information is assumed to be common knowledge.

The CPA Model with incomplete information is typically called the CPA Model and can be stated as follows:

**CPA Model – Incomplete Information Contract**

Maximize $p[S(\underline{q})-\underline{t}]+(1-p)[S(\overline{q})-\overline{t}]$ *(OBJ)*

s.t. $\underline{t}-\underline{\theta}\underline{q}\geq 0$ *(PC-E)*

$\overline{t}-\overline{\theta}\overline{q}\geq 0$ *(PC-I)*

$\underline{t}-\underline{\theta}\underline{q}\geq \overline{t}-\underline{\theta}\overline{q}$ *(IC-E)*

$\overline{t}-\overline{\theta}\overline{q}\geq \underline{t}-\overline{\theta}\underline{q}$ *(IC-I)*

$\underline{t},\overline{t}\geq 0 \qquad \underline{q},\overline{q}\geq 0$

In this model, the principal wants to maximize his or her expected net profit. The *PC-E* and *PC-I* represent the participation constraints for efficient (*E*) and inefficient (*I*) agents, respectively. For the agent to take the contract, s/he must get nonnegative utility. The *IC-E* and *IC-I* represent the incentive constraints for each agent's type, respectively. The goal of these constraints is to ensure that there is an appropriate incentive for the efficient agent to select the contract, designed for the efficient type, and not the inefficient contract; hence *IC-E* must be satisfied. This same reasoning goes for the inefficient type. This model is a nonlinear program (NLP) because of the *S(q)* term in the objective function. However with linear constraints, commercial optimization software BARON (2007) can solve this problem very quickly. The solution to the CPA Model is a menu contract $\{(\underline{t},\underline{q}),(\bar{t},\bar{q})\}$.

Laffont and Martimort (2002) provide the following insights for the CPA Model.

1) The *PC-E* constraint is redundant. It is automatically implied from the *PC-I* and *IC-I* constraints. When adding these two constraints, the result is $(\bar{t}-\bar{\theta}\bar{q})-(\bar{t}-\bar{\theta}\bar{q})\geq 0-(\underline{t}-\bar{\theta}\underline{q})$. Hence $\underline{t}-\bar{\theta}\underline{q}\geq 0$ . With the assumption of $\bar{\theta}-\underline{\theta}>0$ , *PC-E* will never bind. Another explanation can be found in the mimicking concept. Because of the ability of the efficient agent to mimic the inefficient, the *PC-E* constraint is always strictly satisfied.

2) The *IC-I* constraint does not bind. It is irrelevant because the difficulty of asymmetric information comes when the efficient agent is willing to claim that he is inefficient, and not the reverse.

3) The notation $\underline{U}=\underline{t}-\underline{\theta}\underline{q}$ and $\bar{U}=\bar{t}-\bar{\theta}\bar{q}$ represent the information rent for the efficient and inefficient types, respectively.

4) Adding the *IC-E* and *IC-I* constraints gives a monotonicity constraint, which only occurs in the incomplete information case. The relation $\underline{q} \geq \overline{q}$ is a by-product.

5) From the *IC-E* and *IC-I* constraints, there exists lump sum transfer payments $\bar{t}$ and $\underline{t}$. It is enough to set the transfer payment values such that $\overline{\theta}(\overline{q} - \underline{q}) \leq \bar{t} - \underline{t}$ and $\bar{t} - \underline{t} \geq \underline{\theta}(\overline{q} - \underline{q})$.

The binding of constraints can also be shown by Lagrangian relaxation (Fisher 1981) of the CPA Model. The Lagrangian relaxation of the CPA model can be written as:

Maximize $$L(\overline{q}, \underline{q}, \bar{t}, \underline{t}, \lambda, \mu, \gamma) = p[S(\underline{q}) - \underline{t}] + (1-p)[S(\overline{q}) - \bar{t}] + \lambda(\bar{t} - \overline{\theta}\overline{q}) + \mu(\underline{t} - \underline{\theta}\underline{q} - \bar{t} + \underline{\theta}\overline{q}) + \gamma(\bar{t} - \overline{\theta}\overline{q} - \underline{t} + \overline{\theta}\underline{q})$$

s.t. $$\lambda, \mu, \gamma \geq 0$$

Setting the first order condition to zero for $\overline{q}, \underline{q}, \bar{t}, \underline{t}$, there follows;

$$\partial L / \partial \underline{q} : pS'(\underline{q}) - \mu\underline{\theta} + \gamma\overline{\theta} = 0 \qquad I$$
$$\partial L / \partial \overline{q} : (1-p)S'(\overline{q}) - \lambda\overline{\theta} + \mu\underline{\theta} - \gamma\overline{\theta} = 0 \qquad II$$
$$\partial L / \partial \underline{t} : -p + \mu - \gamma = 0 \qquad III$$
$$\partial L / \partial \bar{t} : p - 1 + \lambda - \mu + \gamma = 0 \qquad IV$$

From *III* and *IV*, $\lambda = 1$, which concludes that constraint *PC-I* is tight. Additionally, $\mu = \dfrac{pS'(q) + \gamma\overline{\theta}}{\underline{\theta}}$, which means $\mu > 0$ based upon the key assumptions. Hence, the constraint *IC-E* binds in the CPA Model.

As described in Laffont and Martimort (2002), the Revelation Principle ensures that, without loss of generality, it is possible to restrict the principal to offer simple menus having as many options as the cardinality of type space. The direct revelation mechanism maps type space to allocation space $\{q,t\}$. The processor commits to offer $q(\tilde{\theta})$ and $t(\tilde{\theta})$ if the collector announces his type $\tilde{\theta}$. This means that if one can capture all the type space correctly, i.e. each agent's potential types is a subset of all possible types; the $\{q,t\}$ menu contract is optimal. In the CPA Model, the collector is either efficient or inefficient. With two possible types, the processor can offer an optimal lump sum menu contract with two options to the collector.

More generally in Laffont and Martimort (2002), a complex mechanism is involved with message space, $\mathrm{M}$. Given message $m$ from the agent, the principal requests $\tilde{q}(m)$ and provides $\tilde{t}(m)$. Here the mechanism is a mapping from message space to allocation space. The agent chooses the best message $m^*(\theta)$ such that $\tilde{t}(m^*(\theta)) - \theta\tilde{q}(m^*(\theta)) \geq \tilde{t}(\tilde{m}) - \theta\tilde{q}(\tilde{m}) \;\; \forall \tilde{m} \in \mathrm{M}$. Any allocation rule obtained with this mechanism can be implemented with a truthful direct revelation mechanism. Hence, the truthful direct revelation mechanism for the CPA model is denoted by $\{(\underline{t},\underline{q}),(\bar{t},\bar{q})\}$. By offering this optimal menu contract, the collector will truthfully reveal his or her type by accepting the corresponding contract alternative.

The situation where the agent could be one of more than two types is discussed in the following section.

## 3.2 Multiple Type Principal-Agent (MTPA) Model

The Multiple Type Principal-Agent (MTPA) Model is an extension of the CPA Model and represents cases where the agent could be one of many types. Each of these types is distinguished by a single characteristic - different marginal cost parameter value.

The MTPA Model yields a menu contract alternative for each potential type of agent. The model is adapted from Laffont and Martimort (2002).

Starting with three types of agents, let the marginal costs associated with each agent type be $\theta_1, \theta_2, \theta_3$, and without loss of generality let $\theta_1 \le \theta_2 \le \theta_3$. The smaller subscripts suggest more efficient collectors. The probabilities of a given collector being of type *i* is $p_i$, where $i$ = 1, 2, and 3 and $p_1 + p_2 + p_3 = 1$. The truthful direct revelation mechanism (menu contract) is denoted by $\{(t_1, q_1), (t_2, q_2), (t_3, q_3)\}$.

The MTPA Model for three potential types of agents thus can be written as:

**Full MTPA Model**

| | | |
|---|---|---|
| Maximize | $p_1[S(q_1) - t_1] + p_2[S(q_2) - t_2] + p_3[S(q_3) - t_3]$ | *(OBJ)* |
| s.t. | $t_1 - \theta_1 q_1 \ge 0$ | *(PC-1)* |
| | $t_2 - \theta_2 q_2 \ge 0$ | *(PC-2)* |
| | $t_3 - \theta_3 q_3 \ge 0$ | *(PC-3)* |
| | $t_1 - \theta_1 q_1 \ge t_2 - \theta_1 q_2$ | *(IC-1,2)* |
| | $t_1 - \theta_1 q_1 \ge t_3 - \theta_1 q_3$ | *(IC-1,3)* |
| | $t_2 - \theta_2 q_2 \ge t_1 - \theta_2 q_1$ | *(IC-2,1)* |
| | $t_2 - \theta_2 q_2 \ge t_3 - \theta_2 q_3$ | *(IC-2,3)* |
| | $t_3 - \theta_3 q_3 \ge t_1 - \theta_3 q_1$ | *(IC-3,1)* |
| | $t_3 - \theta_3 q_3 \ge t_2 - \theta_3 q_2$ | *(IC-3,2)* |
| | $t_1, t_2, t_3, q_1, q_2, q_3 \ge 0$ | |

However, this model can be further simplified. Similar to the CPA Model, it can be shown that only the *PC-3* constraint is tight. As discussed in Laffont and Martimort, (2002), the incentive constraints can be classified as *local* and *global*. The local incentive constraints are the ones involved in adjacent agent types (in terms of agent marginal costs in this model), such as constraints *IC-1,2*, *IC-2,1*, *IC-2,3*, and *IC-3,2*. On the other hand, global incentive constraints are the ones for non-adjacent types; constraints *IC-1,3*, and

*IC-3,1* in the MTPA Model. Using the same reasoning as applied to the CPA Model, the more efficient type should not be given any incentive to lie and claim s/he is less efficient. Hence, the incentive constraints that matter are the *IC-1,2*, *IC-1,3*, and *IC-2-3* constraints. When adding the two incentive constraints for the two adjacent types of agents, the quantity constraints are obtained. For example, adding constraints *IC-1,2* to *IC-2,1* yield $q_1 \geq q_2$. Similarly, adding constraints *IC-2,3* and *IC-3,2* yield $q_2 \geq q_3$. Together this yields the monotonicity constraint $q_1 \geq q_2 \geq q_3$. When monotonicity holds, the two local constraints imply a global constraint. Therefore, a simplified model can be written as:

**Simplified MTPA Model – 3 Types**

| | | |
|---|---|---|
| Maximize | $p_1[S(q_1)-t_1]+p_2[S(q_2)-t_2]+p_3[S(q_3)-t_3]$ | *(OBJ)* |
| s.t. | $t_3-\theta_3 q_3 \geq 0$ | *(PC-3)* |
| | $t_1-\theta_1 q_1 \geq t_2-\theta_1 q_2$ | *(IC-1,2)* |
| | $t_2-\theta_2 q_2 \geq t_3-\theta_2 q_3$ | *(IC-2,3)* |
| | $q_1 \geq q_2$ | *(MC-1)* |
| | $q_2 \geq q_3$ | *(MC-2)* |
| | $t_1,t_2,t_3,q_1,q_2,q_3 \geq 0$ | |

This model can be generalized to more than three agents. Let $m'$ be the number of types of agents. Let $k$ be the subscript denoting type index, where $k=1,...,m'$ and let $p_k$ denote the probability of the collector having the $k^{th}$ lowest marginal cost, $\sum_{k=1}^{m'} p_k = 1$. A more general model for $m'$ types can be written as:

**MTPA Model**

Maximize $\sum_{k=1}^{m'} p_k[S(q_k) - t_k]$ *(OBJ)*

s.t. $t_{m'} - \theta_{m'}q_{m'} \geq 0$ *(PC-m')*

$t_k - \theta_k q_k \geq t_{k+1} - \theta_k q_{k+1} \quad k = 1,...,m'-1$ *(IC-k)*

$q_k \geq q_{k+1} \quad k = 1,...,m'\text{-}1$ *(MC-k)*

$t_k, q_k \geq 0 \quad k = 1,...,m'$

The truthful direct revelation mechanism is denoted by $\{(t_1, q_1), (t_2, q_2), ..., (t_{m'}, q_{m'})\}$. With many types, it is necessary to distinguish whether infeasibility occurs when a type is removed. In a large collection network, it is often that a collector defects. In this case, the processor needs to confirm that all the incentives are still enforced. The situation when an agent type is removed, for any reason, must be addressed as follows:

- For the *PC-m'* constraint,

  if the *m'* type is not in the model, the model needs re-solving.

- For the *IC-k* constraint,

  if the $k^{th}$ type is not in the model, check *ex-post* that *IC-k* and *IC-k-1* are satisfied. If not satisfied, the model needs re-solving with the new *IC-k'*, where *k'* is the adjacent larger marginal cost type to *k*.

- For the *MC-k* constraint,

  there is no need to check *ex-post* since it is always satisfied.

The case of re-solving the MTPA Model with new parameters afterwards is explained in Chapter 5 when these steps are repeated until all constraints are satisfied. Suppose that the type *i* collector is removed, the affected constraints are:

*Old*

$t_{i-1} - \theta_{i-1} q_{i-1} \geq t_i - \theta_{i-1} q_i$ (1) (*IC-i-1*)

$t_i - \theta_i q_i \geq t_{i+1} - \theta_i q_{i+1}$ (2) (*IC-i*)

$q_{i-1} \geq q_i \geq q_{i+1}$ (3) (*MC-i-1 & MC-i*)

*New*

$t_{i-1} - \theta_{i-1} q_{i-1} \geq t_{i+1} - \theta_{i-1} q_{i+1}$ (4) (*ICnew*)

$q_{i-1} \geq q_{i+1}$ (5) (*MCnew*)

*Rewritten*

$$(1)+(2): t_{i-1} - t_{i+1} - \theta_{i-1}(q_{i-1} - q_i) + \theta_i (q_i - q_{i+1}) \geq 0 \qquad (6)$$

$$(4): t_{i-1} - t_{i+1} - \theta_{i-1}(q_{i-1} - q_{i+1}) \geq 0 \qquad (7)$$

Hence, it is required to check whether $\theta_{i-1}(q_{i-1} - q_i) - \theta_i (q_i - q_{i+1}) \stackrel{?}{\geq} \theta_{i-1}(q_{i-1} - q_{i+1})$ or not. If yes, then dropping $i$ does not violate any constraint.

For the complete information situation, the principal knows the private information of the agent (type). Hence, there is a contract developed for each of the available types. The solution to the MTPA Model with complete information is $S'(q_1) = \theta_1$, $S'(q_2) = \theta_2$, …, $S'(q_{m'}) = \theta_{m'}$. This solution extends directly from the CPA with complete information contract.

A direct benefit to the processor of solving the contract problem is quantifying the value of information. Knowing net profits in the cases of both complete and incomplete information, s/he can recognize the maximum value (cost) of information. If the processor has knowledge of the collectors' types, s/he can design a contract individually for each collector. S/he should not expend more than the difference between net profits of the complete information and the incomplete information cases to obtain this information (marginal costs).

This chapter has covered the fundamentals of the contract theory applied to my research problem. The other half of the problem is the strategic network problem, which is discussed next in Chapter 4.

# CHAPTER 4

# NETWORK PROBLEM DEFINITIONS & MODELS

The network strategic problem is discussed in this chapter. The network strategic decisions for RPS include: 1) the number and size of the collection and processing sites; 2) the allocation of processing functions to the sites; 3) the routes for products and materials through the potential task network; 4) the modes of transportation used; and 5) the amount of material allocated to each potential end-use, Realff et al. (2000).

A *coupled system* is denoted as a comprehensive representation that simultaneously considers every component (supply, collection, processing, and demand) in the RPS infrastructure to determine both *collection decisions* and *processing decisions*. With a *decoupled system*, an initial examination is made only of the supply and collection stages to determine the collection decisions but then is followed by determination of the processing decisions. I employ an *iterative* strategy with clustering techniques for situations where the first two approaches do not perform well.

The primary objective of the research is to develop and analyze models called Coupled, Decoupled, and Iterated Models as well as to provide suggestions on when to use each formulation. The chapter begins with the Coupled Model in Section 4.1. The Decoupled Model is discussed in Section 4.2, followed by the Iterated Model in Section 4.3. The effectiveness of each approach is measured by the solution quality and computational time, using the more traditional Coupled Model as the baseline. A numerical study is performed in Section 4.4.

## 4.1 Coupled Model

The Coupled Model, which considers the full scope of the decisions in reverse production systems, incorporates decisions for the four stages of RPS infrastructure, (supply, collection, processing, and demand) over multiple periods of time. The mixed integer linear programming (MILP) model for the RPS infrastructure determination problem introduced in Realff et al. (2004) serves as the formulation for the Coupled Model. In words, it can be stated as:

**Maximize: Net Profit**

(sales revenues – fixed costs – storage costs – collection and processing costs – transportation costs);

**Subject to: Conservation of Flow between Sites**

(on material consumed and produced by the tasks located at sites);

**Upper and Lower Bounds**

(on storage, transportation, and processing of materials at sites);

**Logical Constraints on Sites**

(i.e., the need to open a site before allowing tasks to be located there).

Reviewing the MILP from Realff et al. (2004) provides the starting point for developing the Decoupled and Iterated Models. Because the Coupled Model produces optimal solutions, it is utilized as the baseline in comparing the other two models. The indices are: $i$ represents sites, $j$ represents material types, $m$ represents transportation modes, $p$ represents the process types, $q$ represents the replications of recycling tasks, and $t$ represents the time periods. The following superscripts are used: $c$ stands for collection, $r$ stands for storage, $s$ stands for site, $h$ stands for transportation, and $d$ stands for material sales.

The continuous decision variables are:

1) $M_{ijt}$, the amount of type $j$ material collected at site $i$ at time $t$;
2) $S_{ijt}$, the amount of type $j$ material stored at site $i$ at time $t$;
3) $H_{iji'mt}$, the amount of type $j$ material shipped from site $i$ to $i'$ using transportation mode $m$ at time period $t$;
4) $D_{ijt}$, the amount of type $j$ material sold at site $i$ during time period $t$; and
5) $\xi_{ipt}$, the extent of process $p$ performed at site $i$ in period $t$.

The binary decision variables include:

1) $y_i^{(c)}$ equals 1 if collection is to be performed at site $i$ and equals 0 otherwise;
2) $y_{ii'm}^{(h)}$ equals 1 if shipment is allowed between site $i$ and site $i'$ of transportation mode $m$ and equals 0 otherwise;
3) $y_{ipq}$ equals 1 if replica $q$ of process $p$ is to be allowed at site $i$ and equals 0 otherwise;
4) $y_{ij}^{(r)}$ equals 1 if storage of material $j$ is to be allowed at site $i$ and equals 0 otherwise;
5) $y_{ij}^{(d)}$ equals 1 if material $j$ is allowed to be sold at site $i$ and equals 0 otherwise; and
6) $y_i^{(s)}$ equals 1 if site $i$ is to be opened and equals 0 otherwise.

The parameters for the MILP employ the following notations:

$P_{ijt}$, the price of selling type $j$ material at site $i$ at time $t$;
$K_i^{(r)}$, the unit cost per time period for storage at site $i$;
$K_i^{(c)}$, the unit cost per time period for collection at site $i$;
$K_{ip}$, the unit flow cost per time period for process $p$ at site $i$;
$K_{ii'm}^{(h)}$, the unit transportation cost per distance from sites $i$ to $i'$ of mode $m$;
$b_{ii'm}$, the distance from sites $i$ to $i'$ by transportation mode $m$;
$\rho_{jp}$, the proportion of type $j$ material consumed by process $p$;
$\rho'_{jp}$, the proportion of type $j$ material produced by process $p$;
$f_i^{(s)}$, the fixed opening cost of site $i$;
$f_i^{(r)}$, the fixed storage cost of site $i$;

$f_i^{(c)}$, the fixed material collecting cost of site *i*;

$f_{ip}$, the fixed cost of process *p* at site *i*;

$f_{ii'm}^{(h)}$, the fixed cost of transportation from sites *i* to *i'* by transportation mode *m*;

$\varepsilon_{ijt}^{(c)}$, the maximum collection capacity of material type *j* at site *i* at time period *t*;

$\varepsilon_{ij}^{(d)}$, the maximum amount of material type *j* that can be sold at site *i* at any time period;

$\varepsilon_{ij}^{(r)}$, the maximum amount of material type *j* that can be stored at site *i* at any time;

$\varepsilon_{ii'm}^{(h)}$, the maximum amount of material that can be shipped from sites *i* to *i'* by transportation mode *m*;

$\varepsilon_{ipt}$, the maximum amount of material that process *p* produces at site *i* at time *t*;

$\alpha_i^{(r)}$ equals 1 if storage is allowed at site *i*, and 0 otherwise;

$\alpha_{ij}^{(d)}$ equals 1 if material j can be sold at site *i*, and 0 otherwise;

$\alpha_{ii'm}^{(h)}$ equals 1 if shipment is allowed between site *i* and site *i'* of transportation mode *m*, and 0 otherwise;

$\alpha_{ip}$ equals 1 if process *p* is allowed at site *i*, and 0 otherwise;

$\alpha_i^{(c)}$ equals 1 if collection is allowed at site *i*, and 0 otherwise;

Utilizing the above notations, the RPS strategic infrastructure planning problem is posed as follows.

**Coupled Model**

Maximize (Objective) — Maximize (Net Revenue) (1)

$$\sum_t \sum_i \sum_j P_{ijt} D_{ijt}$$ Sales revenue

$$-\sum_i (f_i^{(c)} y_i^{(c)} + f_i^{(s)} y_i^{(s)} + f_i^{(r)} y_i^{(r)})$$ Fixed cost

$$-\sum_i \sum_p \sum_q f_{ip}\, y_{ipq} - \sum_i \sum_{i' \neq i} \sum_m f_{ii'm}^{(h)} y_{ii'm}^{(h)}$$

$$-\sum_t \sum_i \sum_j K_i^{(r)} S_{ijt}$$ Storage cost

$$-\sum_t \sum_i \sum_j K_i^{(c)} M_{ijt} - \sum_t \sum_i \sum_p K_{ip} \xi_{ipt}$$ Collection and processing costs

$$-\sum_t \sum_i \sum_j \sum_{i' \neq i} \sum_m K_{ii'm}^{(h)} b_{ii'm}^{(h)} H_{iji'mt}$$ Shipping costs (1A)

Subject to:

$$S_{ijt} = M_{ijt} + S_{ijt-1} - \sum_{\text{supplies}} \sum_{m} H_{ijimt} + \sum_{i' \neq i} \sum_{m} H_{i'jimt} \qquad \forall i, j, t \quad \text{Conservation of flow} \quad (2)$$

$$-\sum_{i' \neq i} \sum_{m} H_{iji'mt} - \sum_{\text{demands}} \sum_{m} H_{ijimt} + \sum_{p} \rho'_{jp} \xi_{ipt}$$

$$-\sum_{p} \rho_{jp} \xi_{ipt}$$

| | | | |
|---|---|---|---|
| $y_i^{(c)} \leq y_i^{(s)}$ | $\forall i$ | Logical constraints | (3) |
| $y_{ipq} \leq y_i^{(s)}$ | $\forall i, p, q$ | | (4) |
| $y_i^{(r)} \leq y_i^{(s)}$ | $\forall i$ | | (5) |
| $y_{ij}^{(d)} \leq y_i^{(s)}$ | $\forall i, j$ | | (6) |
| $y_i^{(c)} \leq \alpha_i^{(c)}$ | $\forall i$ | | (7) |
| $y_{ipq} \leq \alpha_{ip}^{(c)}$ | $\forall i, p, q$ | | (8) |
| $y_i^{(r)} \leq \alpha_i^{(r)}$ | $\forall i$ | | (9) |
| $y_{ij}^{(d)} \leq \alpha_{ij}^{(d)}$ | $\forall i, j$ | | (10) |
| $y_{ii'm}^{(h)} \leq \alpha_{ii'm}^{(h)}$ | $\forall i, i' \mid i \neq i', m$ | | (11) |
| $y_{ipq+1} \leq y_{ipq}$ | $\forall i, p, q$ | | (12) |
| $M_{ijt} \leq \varepsilon_{ijt}^{(c)} y_i^{(c)}$ | $\forall i, j, t$ | Capacity constraints | (13) |
| $H_{iji'mt} \leq \varepsilon_{ii'm}^{(h)} y_{ii'm}^{(h)}$ | $\forall i, i' \mid i \neq i', m, t$ | | (14) |
| $S_{ijt} \leq \varepsilon_{ij}^{(r)} y_{ij}^{(r)}$ | $\forall i, j, t$ | | (15) |
| $D_{ijt} \leq \varepsilon_{ij}^{(d)} y_{ij}^{(d)}$ | $\forall i, j, t$ | | (16) |
| $\xi_{ipt} \leq \sum_{q} \varepsilon_{ipt} y_{ipq}$ | $\forall i, p, t$ | | (17) |
| | | Variable constraints | (18) |
| $M_{ijt}, S_{ijt}, H_{iji'mt}, D_{ijt}, \xi_{ipt} \geq 0$ | $\forall i, i' \neq i, p, m, t$ | | |
| $y_i^{(c)}, y_{ii'm}^{(h)}, y_{ipq}, y_i^{(r)}, y_{ij}^{(d)}, y_i^{(s)} \in \{0,1\}$ | $\forall i, i' \neq i, p, m, q$ | | (19) |

Solutions to this model yield mathematically optimal strategic infrastructure design decisions. Nevertheless, there are shortcomings with the Coupled formulation. First, and most importantly, there is a significant amount of uncertainty that is not captured well by this deterministic model. Good long-term decisions are desired to provide comfort with a significant commitment of resources and effort, as well as operational viability. To address this problem, Realff et al. (2004) propose a robust optimization model formulation and Hong et al. (2006) implement a robust scenario-based RPS design. Secondly, as the problem size becomes large scale, the computational effort required to solve it becomes unacceptable. To address this problem, I propose a Decoupled Model which finds “good” but not necessarily globally optimal solutions with computationally tractable solution requirements. Case studies have shown that the Decoupled Model provides reasonably high-quality solutions. In the next section, decomposition principles are applied to the Coupled Model to develop the Decoupled Model.

## 4.2 Decoupled Model

There are three motivations for developing a model that decouples the collection decisions from the processing decisions. First, it is often the case in practice that the collection is planned and implemented separately from the processing infrastructure, and so the Decoupled Model captures this situation. Second, according to data from the Hong et al. (2006) case study, when the focus is on a regional area, typically the RPS processing strategic decisions do not have a significant impact on the RPS collection decisions. Third, the Coupled Model becomes unmanageably large for regional scale problems and the computation time prohibitive. These factors stimulated my development of a decomposition approach for RPS strategic planning.

Note that the objective function component (1A) of the Coupled Model can be decomposed into three components, as shown below.

$$\sum_t \sum_i \sum_j \sum_{i' \neq i} \sum_m K_{ii'm}^{(h)} b_{ii'm}^{(h)} H_{iji'mt} = \sum_t \sum_{i|\text{collection sites}} \sum_j \sum_{i' \neq i|\text{sources}} \sum_m K_{ii'm}^{(h)} b_{ii'm}^{(h)} H_{iji'mt} \tag{1A1}$$

$$+\sum_t \sum_{i|\text{processing sites}} \sum_j \sum_{i' \neq i|\text{collection sites}} \sum_m K_{ii'm}^{(h)} b_{ii'm}^{(h)} H_{iji'mt} \tag{1A2}$$

$$+\sum_t \sum_{i|\text{demand sites}} \sum_j \sum_{i' \neq i|\text{processing sites}} \sum_m K_{ii'm}^{(h)} b_{ii'm}^{(h)} H_{iji'mt} \tag{1A3}$$

The inbound transportation to the collection points, (1A1), consists of passenger vehicles, less-than-truckload trucks, and occasionally full-truckload trucks. The outbound transportation from collection sites to the processing sites, (1A2), and from the processing points to end-users, (1A3), is frequently full-truckload trucks. As a result in (1A1), there is a much more expensive collection transportation cost than the post-collection transportation costs of (1A2) and (1A3).

However, the single most important reason to consider a decomposed model is the complexity of the large-scale problem. For example, a problem structure with a high resolution that focuses on cities instead of counties or regions of the U.S. cannot be solved in a reasonable computational time. As a result, focusing on separate supply and collection phases is conceptually appealing. The Decoupled Model formulation for the collection phase has the following modifications to the Coupled Model:

- Consideration only of terms maximizing net profit composed of the fixed costs of collection sites, storage costs, collection costs, and transportation costs to and from collection sites
- In the net conservation of flow constraints (2), elimination of the last four summation terms: the transportation from-to processing site terms and the production and consumption at processing site terms (which are done at all indexed points).
- Elimination of the logical constraints (4), (6), (10), (11), and (12).

- Elimination of the capacity constraints (16) and (17)
- Removal of the last two terms of the variable restrictions constraints (18)
- Elimination of the third and fifth terms of the variable restrictions constraints (19)

Since these modifications remove the processing and demand stages from the RPS infrastructure, it is possible to focus only on the collection infrastructure (supply and collection stages). As a result, the maximization of net profit is equivalent to minimizing cost.

The strategic decisions made by the Decoupled Model for the collection phase include the location, number, and size of the collection sites, the modes of transportation between supply and collection sites, and the flows of products and materials from the supply to collection sites. Once these collection decisions are determined, the processing decisions are easily obtained (provided the processing information is available) simply by substituting the values for the $y_i^{(s)}$ solutions obtained by solving the Decoupled Model for the collection phase into the Coupled formulation. From numerous research instances, solving the Coupled formulation containing these fixed collection decisions can be performed relatively quickly. Due to the specific RPS Model structure, the Decoupled collection solution values are always feasible when the integer variables are fixed in the Coupled Model.

The Coupled Model never gives a worse solution than the Decoupled Model because of its integrated decision structure. Consequently, one can view the Decoupled formulation as utilizing local structures with a trade off between solution quality and execution time. From case study investigations, the solution quality of the Decoupled Model is generally excellent and obtained with lower computational requirements. This is consistent with the reality of current RPS infrastructures where the coupling between collection decisions and processing decisions is small. Municipalities often take on the former responsibility while the latter is often true for the private sector whose location is driven by the existing infrastructure reflecting the demand for collection services.

Nevertheless, in a very small number of specific cases, it was found that the Decoupled Model may not yield a satisfactory result. The researcher's experience demonstrates that it is not possible to entirely concentrate on the mathematically optimal objective function value in making comparisons. In these cases, the optimal solution structures differ between Coupled and Decoupled Model solutions, even with a similar objective function value. One example of these specific cases is when the problem has sites located in clusters. To solve this problem, a new formulation called "Iterated Model" is introduced and will be discussed in the next section.

## 4.3 Iterated Model

As pointed out in the previous section, there can be cases where the Decoupled Model may not perform well. In this subsection I will illustrate these limitations with small examples. Unfortunately, these problem situations may occur in many actual RPS infrastructure design problems.

The first case of concern is when the collection sites and processing sites are not mixed uniformly throughout the region; that is, when there are clusters of sites. I illustrate this limitation with a small example as depicted in Figure 4.1a). In its optimal solution the Coupled Model opens the Collection Site 2 (CS2) and Processing Site 2 (PS2) whereas the Decoupled Model solution opens all three collection sites and PS2.

Even though the objective function value of the Decoupled Model solution has a small deviation of only 1.5% from the value of the solution obtained by the Coupled Model, it clearly has a different collection infrastructure. This deviation could be much larger with different parameters values for site opening costs and transportation costs. With the Decoupled Model, instead of opening a shared collection and processing at the center between two clusters, there are individual collection and processing sites for each cluster. This small example could be generalized to the case of many clusters of collection sites in the big cities on the coast but only processing sites in the central part of the country. In these types of situations, the solutions obtained by the Decoupled Model do not perform well.

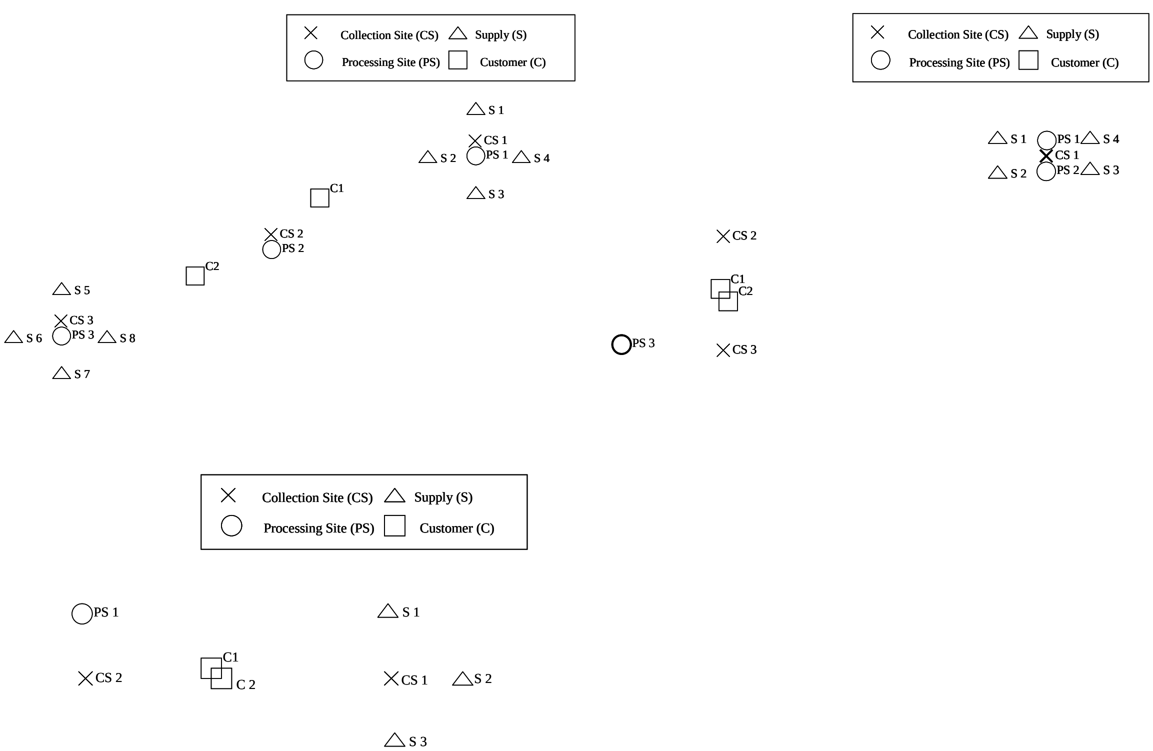



Figure 4.1: Examples of Decoupled Model Limitation – a) top-left: Clusters, b) top-right: Significant capacity difference, c) bottom-left: Transportation cost difference

Another dysfunctional case is when the capacity of collection sites and processing sites are significantly different. If there are some very high capacity collection sites in the region, but only a few low capacity processing sites, then there is a need to send the collected materials to be processed in other regions. A small example shows this limitation in Figure 4.1b). Here the CS1 and PS3 have high capacity whereas PS1 and PS2 do not have enough capacity to serve the processing demand in the cluster. The optimal solution obtained by the Coupled Model opens CS2, CS3, and PS3 whereas the solution to the Decoupled Model opens CS1, CS2, CS3, and PS3 with a difference in objective function values of 3%. One can see that in the Decoupled solution there is an unnecessary collection site (CS1) that does not need to be present in the optimal solution. When there are significant differences between the collection and production site capacities, use of the Decoupled Model is not recommended.

Finally, when the relative transportation costs between the supply-collection sites and the collection-processing sites are widely dissimilar, the Decoupled Model does not perform well. In the Figure 4.1c) example with the collection-processing site transportation costs per distance four times those of the supply-collection site transportation costs, the Decoupled Model determines a different infrastructure solution than the Coupled Model. The Decoupled Model solution opens CS1, CS2, and PS1 while the Coupled Model solution opens CS2 and PS1. Moreover, there is a difference in solution objective values of 2.3%, but, with different parameter values, the solution value differences could potentially be very large. However, this case corresponds to the particular scenario where the consumer essentially would be bearing the cost of supply to the collection transportation which would cause the cost structure of transportation to shift to a low local cost and high long distance cost pattern for the RPS, rather than the expected high local cost and low long distance cost on a per item or lb basis. This situation will arise if the model represents an entity that considers only the capital and operating costs of the collection and processing centers as well as the transportation between them and the end market, but fails to consider the consumer cost. In the current RPS operation, this is a likely scenario as consumers are not usually compensated for bringing items to collection centers. But if curbside recycling or drop-off bins are utilized, it would not apply.

For commonly found situations such as the three cases described above, I propose the Iterated Model. Although some data manipulations are needed for this model, they are simple and do not require very complicated coding. As an alternative, though, one could attempt solving the Coupled Model with associated high computation time for large-scale problems.

In the first step of the Iterated Model, every collection site is assigned to a cluster, although it is possible to have a single site in a cluster if needed. Exclusive clustering is desired such that each site belongs to only one particular cluster. Cluster assignments can be done geographically (by county, by region or by state). For example, one can directly form one cluster of the metro-area of a major city as a geographical consideration. One can also create clusters mathematically by using fast minimum spanning tree (MST) algorithms, Ahuja et al. (1993). Once a MST is determined, the $n$ longest arcs can be removed to obtain $n+1$ clusters. There are many other algorithms for the cluster determination, e.g., Rasmussen (1992). Since accuracy in this approach is not a particular concern, a fast method is preferred.

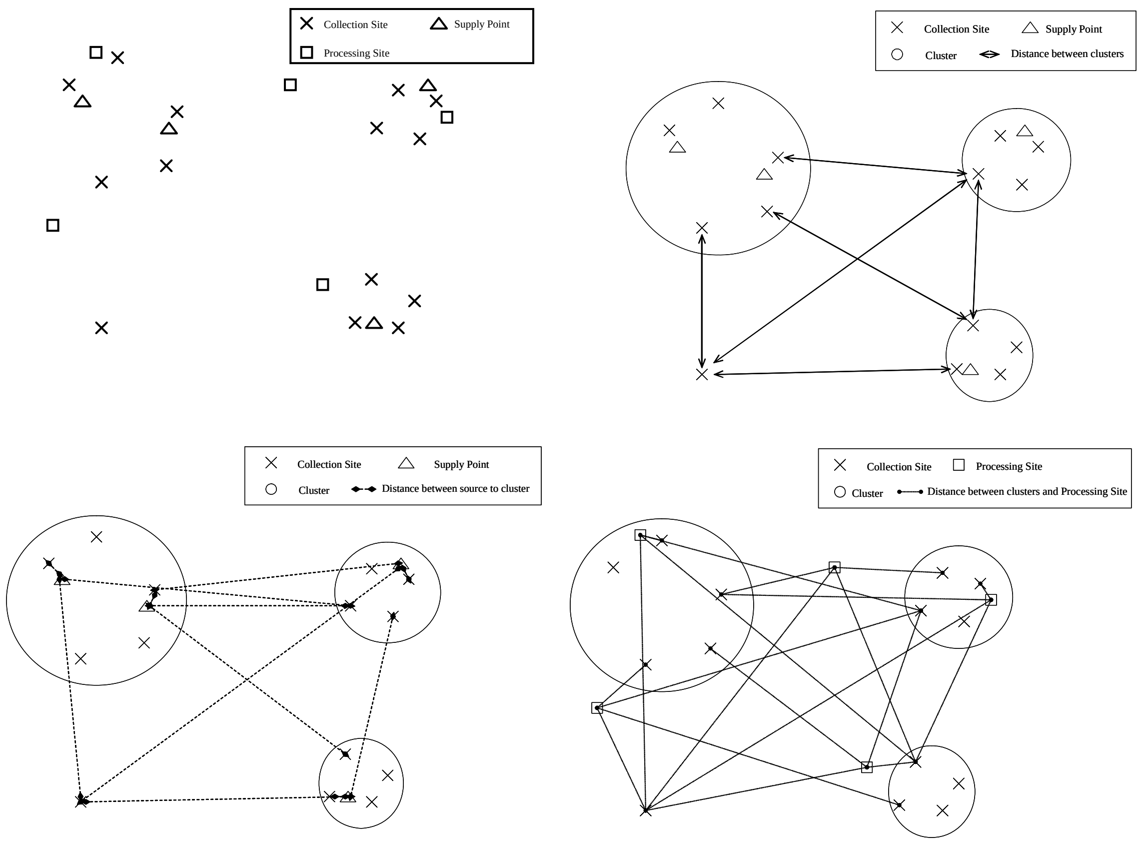



Figure 4.2: Iterated Model Data - a) top-left: Regular data, b) top-right: Cluster assignment and distances between clusters, c) bottom-left: Distances between suppliers and clusters, and d) bottom-right: Distances between processing sites to clusters

The Iterated Model uses two sets of data: regular and cluster data. Figure 4.2 describes the two data types. Figure 4.2a) represents the regular data including the supply and collection sites. Figure 4.2b) represents the collection sites cluster assignment and the distance between clusters. Let $C_I$ represent the cluster $I$ and $C_J$ represent the cluster $J$. Also, let $i$ and $j$ index the sites in $C_I$ and $C_J$ respectively. Let $dc_{IJ} = \min\{d_{ij} \mid \forall i \in C_I \text{ and } \forall j \in C_J\}$ represent the distance between cluster $I$ and cluster $J$. Figure 4.2c) shows the distances between supplies and clusters. Let $s$ be the index for supply points. Then, the distance between a supply $s$ and a cluster is calculated from $ds_{sI} = \min\{d_{si} \mid \forall i \in C_I\}$. Figure 4.2d) displays the distances between clusters (collection) and processing sites. Let $k$ be the index for the processing sites. Then, the distance between a cluster and a processing site is calculated from $dp_{Ik} = \min\{d_{ik} \mid \forall i \in C_I\}$. The distances between clusters, distances between supply and clusters, distances between supply and clusters, the minimum site opening cost of all sites in each cluster, and the total collection capacities in each cluster constitute the cluster data.

The Iterated Model is composed of two submodels, namely, the lowerbound model (*LM*) and the upperbound model (*UM*). The *LM* is a RPS Coupled Model that utilizes the regular data. The *UM* is a modifications of the Coupled Model that utilizes the cluster data. The *UM* has the following modification to the RPS Coupled Model:

- Let $i = 1, 2, ..., N_{collect}, ..., N_{total}$, where $N_{collect}$ represents the number of collection clusters. There are $N_{total} - N_{collect}$ processing sites, which is the same as the regular data.
- For each cluster, assign a nonnegative integer variable, $n_i$ to denote the minimum required sites.
- Introduce a new variable, $C_i^{\max}$, which represents the maximum capacity of a site within the cluster *i*.
- Include the minimum sites needed to be open for feasibility in the cluster constraint of $n_i \geq \sum_t \sum_j M_{ijt} / C_i^{\max} \quad \forall i = 1, ..., N_{collect}$.

- In (1), replace the term $-\sum_i f_i^{(s)} y_i^{(s)}$ with $-\sum_{i=1}^{N_{collect}} n_i f_i^{(s)} - \sum_{i=N_{collect}+1}^{N_{total}} f_i^{(s)} y_i^{(s)}$. This incorporates the minimum sites needed to be open within the cluster *i* and aims to help the convergence of upperbound and lowerbound.

At each iteration, the *UM* and *LM* models are solved. Let *Z* be the optimal objective function value (total net profit) of the solution for the Iterated Model. Given an opening infrastructure (decisions of what sites to be opened) $\pi$, then $Z(\pi)$ obtained with the original data is no better than the solution value obtained using the cluster data. This holds true by construction since the transportation costs, site opening costs, and capacity of the cluster data are lower than in the formulation with the original data. From this point forward, I will refer to the objective function value of the solution to the *LM* at iteration *i* using the original data as the lower bound model value, $LB_i$, and the objective function value of the *UM* at iteration *i* using the cluster data as the upper bound model value, $UB_i$. Let $LB_i^*$ represent the current best lower bound model objective value.

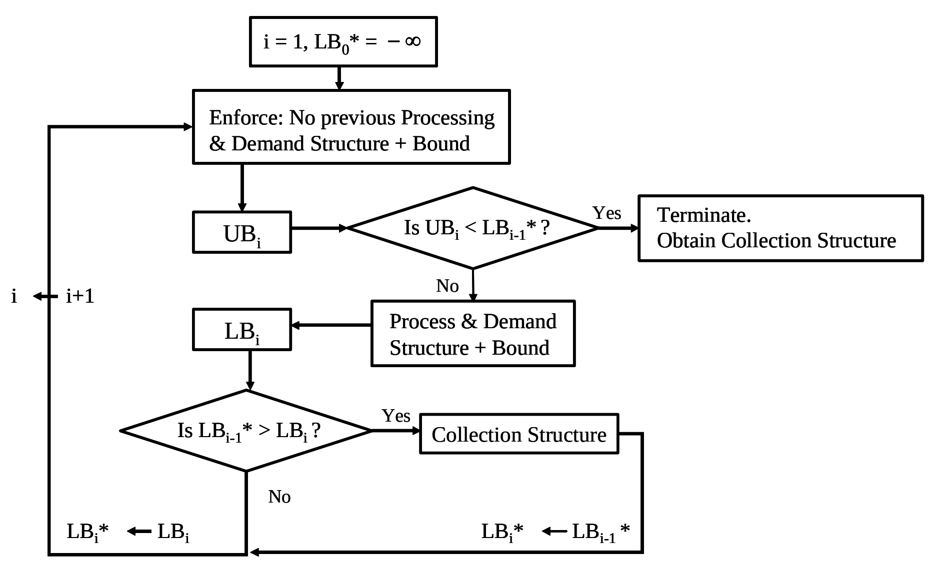



Figure 4.3: Iterated Model Steps

Figure 4.3 illustrates the steps in the Iterated Model approach. At each iteration, the *UM* is solved first to obtain the opened collection clusters, optimal processing and demand infrastructures. These processing and demand structures are fixed in the *LM* to solve for the optimal collection infrastructure (what collection sites to be opened). At the end of iteration, $UB_i$ and $LB_i$ are obtained. The current best solution, $LB_i^*$, is also updated at each iteration. The iterations terminate only when the $UB_i$ has higher cost than the $LB_{i-1}^*$. If not, another iteration is executed. If a better (higher) $LB_i$ than the $LB_{i-1}^*$ is obtained, the $LB_i^*$ is updated to this value. If not, the same current best value is used. The crucial step is to enforce that no previously obtained processing infrastructure configuration can be allowed in the solution of the next $UB_{i+1}$ model. Additionally, a bound on collection is enforced. Each collection cluster comprises fixed opening and variable costs. With the flow information from the *LM*, a (lower) bound can be inserted into the next *UM*. The Iterated Model iterations continue until the termination condition is satisfied. With a finite number of sites, it is guaranteed to terminate after a finite number of iterations.

To reduce iterations, additional constraints can be inserted at the enforcing steps to reflect a specific problem structure. Examples of these additional constraints for the research case study include the type and the nearness of the collection and processing sites. The types of sites include governmental, or non-profit, or large commercial. Each type has a different site opening cost, capacity, and variable cost. The nearness specification can be specified subjectively. In the computational study reported in Section 4.5, when sites are no more than 50 miles apart, it is assumed they are near and belong in the same cluster. When a certain type of collection site does not belong to the optimal solution, it is extended to other collection sites in the proximity of the same type to eliminate symmetric solutions as a group. One can add logical constraints to force some particular sites not to open, which can drastically reduce the number of iterations.

The $UB_i$ is non-increasing between iterations. This is trivial because before each $UB_i$ is solved, at least one elimination constraint (no duplicate process and demand infrastructures) is added. Figure 4.4 shows an example of the solution values over iterations. At iteration $i$, if the $LB_i$ turns out to be the new current best value $LB_i^*$, $UB_{i+1}$ and $LB_{i+1}$ are next solved to find the same current best value. At this next iteration, the $UB_{i+2}$ crosses the current best value to terminate the algorithm, thus obtaining the collection structure from the current best solution, which is the solution of the $LB_i$ model meeting the termination criterion.

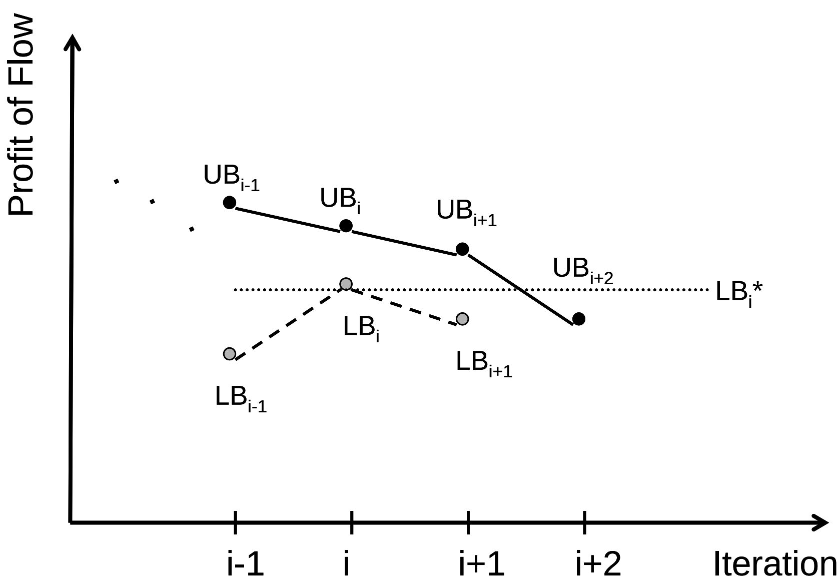



Figure 4.4: Example of Bound Behavior for the Iterated Model over Several Iterations

With a finite number of sites, the Iterated Model always converges. The current best solution is non-decreasing and the $UB_i$ is non-increasing between iterations. As a result, the Iterated Model yields an optimal solution.

This chapter has discussed the development of the three models. To validate the two proposed models, Decoupled and Iterated, it is necessary to implement numerical studies. This research has used the Coupled Model as the baseline in evaluating the performances. These implementations are discussed in the subsequent section.

## 4.4 Numerical Study

This part gives numerical studies for the purposes of validation and verification. The Decoupled Model is applied to the case study data of Hong et al. (2006), in section 4.4.1. Subsequently, the Iterated Model is implemented to the generated examples with clusters in section 4.4.2.

### 4.4.1 Decoupled Model Example

The Decoupled Model is implemented to the E-Scrap in the State of Georgia Case Study, Hong et al. (2006). Rather than repeat the details of the case study, I will only discuss the performance of the Decoupled Model in terms of solution quality and time. Using the Windows 2000-based personal computer with Pentium 1.80 GHz with 1GB RAM, the Coupled and Decoupled Models are run with C++ program and CPLEX 9.0 (2007) for the optimization software. MS-Access was used to store and manage the case study database. The Decoupled Model performance, with the 16 scenarios, can be summarized in Figure 4.5 and Table 4.1.

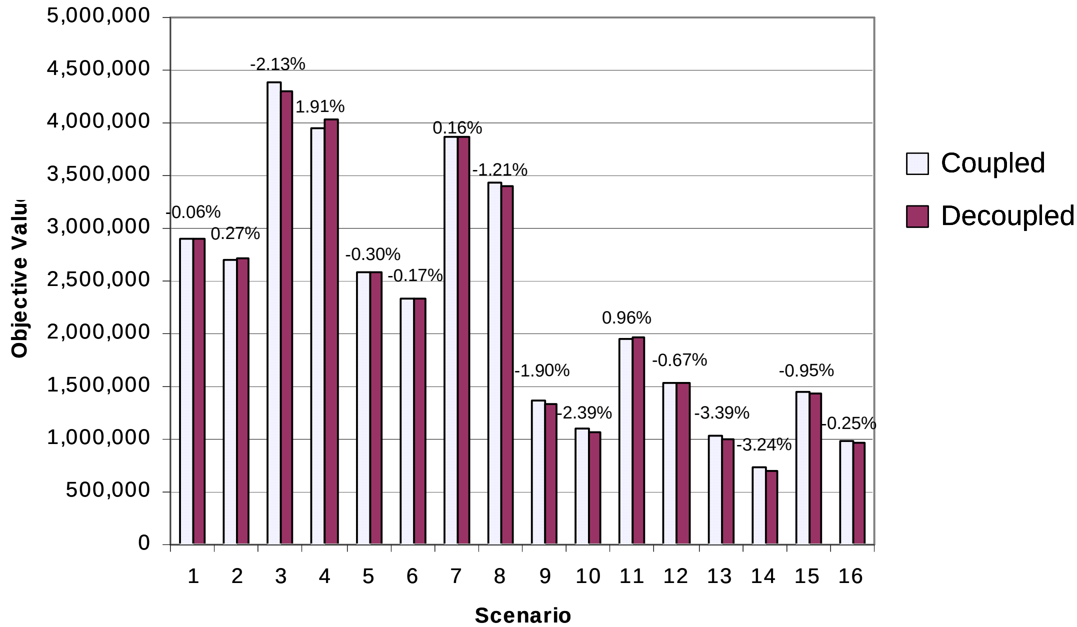



Figure 4.5: Comparison of Decoupled Model solution versus Coupled Model Solution Net Profits for 16 Case Study Scenarios

The solution quality performance of the Decoupled Model is excellent when compared to the baseline values obtained by the Coupled Model. For all 16 scenarios, the optimal net profits of the Decoupled Model solution are no larger than 3.39% of those associated with the optimal solution of the Coupled Model. Importantly, half of the scenarios optimal solutions are within 1.00%. Given the uncertainty and ambiguity in the data, this performance would appear reasonable.

Table 4.1 reports the solution time required for the Coupled and Decoupled Models for each of the sixteen scenarios. The Decoupled Model requires a significant reduction in computational time. For all 16 scenarios, there is at least a 23% saving from the baseline Coupled Model. Twelve of the sixteen scenarios have time reductions of at least 50%. From the research experience (with other problems as well as the ones reported in this case study), a greater time savings is observed with larger problem sizes.

Table 4.1: Comparison of Decoupled vs. Coupled Models Solution Time

| Scenario | Coupled Model (Sec) | Decoupled Model (Sec) | Time Difference (Sec) | % Time Reduction |
|---|---|---|---|---|
| 1 | 232 | 103 | 129 | 55.6% |
| 2 | 147 | 40 | 107 | 72.8% |
| 3 | 64 | 49 | 15 | 23.4% |
| 4 | 52 | 37 | 15 | 28.8% |
| 5 | 261 | 35 | 226 | 86.6% |
| 6 | 176 | 38 | 138 | 78.4% |
| 7 | 64 | 40 | 24 | 37.5% |
| 8 | 67 | 37 | 30 | 44.8% |
| 9 | 3,338 | 370 | 2,968 | 88.9% |
| 10 | 2,267 | 455 | 1,812 | 79.9% |
| 11 | 265 | 46 | 219 | 82.6% |
| 12 | 561 | 186 | 375 | 66.8% |
| 13 | 4,030 | 996 | 3,034 | 75.3% |
| 14 | 6,349 | 827 | 5,522 | 87.0% |
| 15 | 428 | 154 | 274 | 64.0% |
| 16 | 1,404 | 495 | 909 | 64.7% |

### 4.4.2 Iterated Model Example

I am also concerned about many actual situations where sites may be clustered together in regions. To test the solution quality and computation time of the Iterated Model for this situation, examples with a cluster structure, known to cause potential problems for the Decoupled Model, were generated. The problem inputs include the number of clusters, maximum number of sites allowed in a cluster, and the size of the dimensional plane. For this example, a random number generator utilizing a uniform distribution is used to position the site locations. For the clusters, the generator first determines the cluster center location, then the number of sites in the cluster is generated within a 25 mile radius from the center. The site locations are enforced to satisfy the exclusive clustering condition such that each site belongs to only one cluster. With these site locations and other parameter specifications, the cluster structure examples are then obtained.

For the tests, examples of small (1000x1000 mile$^2$, 40 collection and 15 processing sites), medium (1000x1000 mile$^2$ , 100 collection and 30 processing sites), and large (2000x2000 mile$^2$ , 320 collection and 80 processing sites) problem sizes were generated. Table 4.2 reports the results which suggest that as the problem size grows larger, the Iterated Model provides larger advantages in computational requirements. In the large problem, the Iterated Model outperforms the Coupled Model in solution time by more than 50%.

Table 4.2: Iterated Model to Generated Examples

| | Coupled Model | | Iterated Model | | |
|---|---|---|---|---|---|
| **Description** | **Solution Objective Function Value** | **Solution Computa-tion Time (sec)** | **Solution Objective Function Value** | **Solution Computa-tion Time (sec)** | **Solution Time Difference (sec)** |
| small problem | $331,866 | 1 | $337,837 | 7 | +6 |
| medium problem | $1,536,782 | 15 | $1,532,122 | 29 | +14 |
| large problem | $991,638 | 1823 | $995,285 | 796 | -1027 |

In this chapter, I have developed the Decoupled and Iterated Models. Details of Coupled, Decoupled, and Iterated Models are discussed with numerical studies. Although the Decoupled Model does not provide optimal solutions, it does provide good solutions with significant execution time reduction in most cases. The Iterated Model gives the optimal solution with more advantages in computational requirements as the problem size grows.

All of the strategic network design models address the four stages of RPS. However, in the research problem, the contract model and network models are connected at the collection and processing stages.  The incorporation of the contract and network model is proposed in the following chapter.

# CHAPTER 5

# CONTRACT/NETWORK LUMP SUM MODELS

This chapter presents the contract and network lump sum models. Two approaches for Lump Sum Models are investigated. The first is a deterministic mathematical programming approach which is discussed in this chapter. The second is a stochastic programming approach which will be presented in Chapter 6.

This chapter begins with a discussion of collector types in Section 5.1, followed by a non-regional and regional contract model discussion in Section 5.2. A Sequential Model is applied to the regional model in Section 5.3 which is followed by a Simultaneous Model in Section 5.4. Section 5.5 presents a numerical study of all the models in this chapter. This chapter concludes with a discussion of the advantages of the Simultaneous Model over the Sequential Model.

## 5.1 Collector Type

I assume that collectors are categorized into types, based on their marginal costs. In economic terms, marginal cost is defined as the change in total cost that arises when the quantity produced (or purchased) changes by one unit. Mathematically, it can be written as $\text{marginal cost} = \dfrac{\partial \text{ total cost}}{\partial \text{ quantity}}$. Marginal cost is represented by $\theta$, in units of $/lb for my studies where I am concerned with the cost of collecting certain quantities of material. This marginal cost usually has an embedded profit margin. As happens in numerous supply chains, the more powerful member drives the less dominant one(s) to sell at cost. Hence, some percentage of profit has to be designed into the collector's marginal cost. The typical marginal cost curve, which has a "u-shape" (convex), can be broken into three regions, as shown in Figure 5.1.

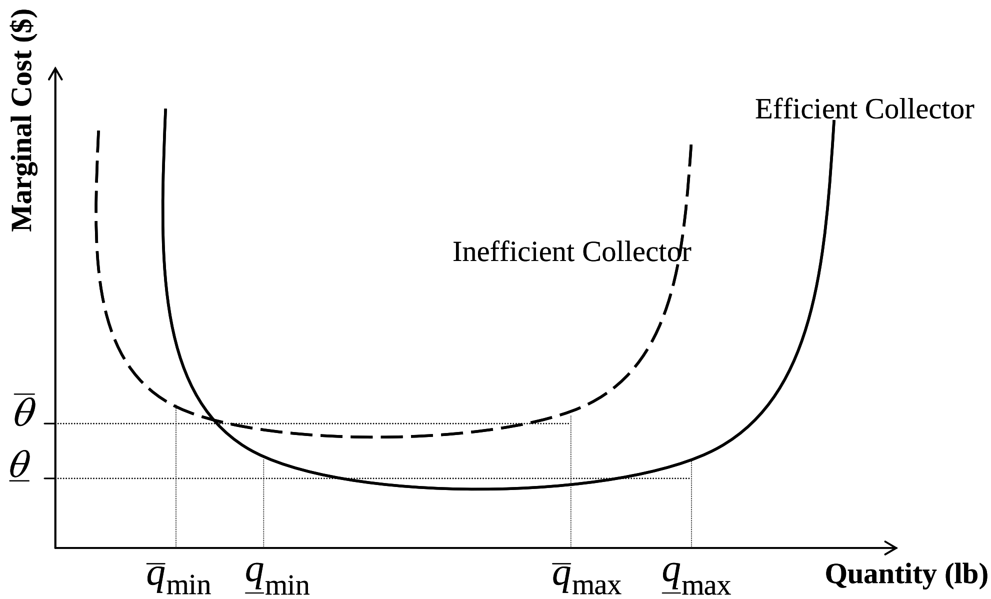



Figure 5.1: Marginal Cost Curve and Constrained Operational Regions

The first region, in the small quantity domain, has a high marginal cost drop as the start up costs are typically very high. The second region, which contains the optimal marginal cost, is a relatively flat section. Because this section is where the collector operates, constraints are placed so that operational quantity is limited within this section. Finally the third region, in the large quantity domain, is where the marginal cost rises rapidly. Reasons for the rapid climb could be that the plant capacity is reached (passed) or that new technology needs to be purchased.

This categorization of collectors by marginal costs is implemented throughout the discussion of lump sum and variable volume models.

## 5.2 Non-Regional and Regional Contract Models

I propose two models: a Non-Regional and Regional Model. The Non-Regional is the initial model that only considers two collector types. On the other hand, the Regional Model deals with multiple types with incorporation of additional factors to determine the type.

The Non-Regional model assumes that the collector is either efficient or inefficient. Additional constraints are added to the CPA Model to reflect that each collector wants to be in the operational domain. The contract model can be written as:

**Non-Regional Principal-Agent (NRPA) Model**

| | | |
|---|---|---|
| Maximize | $p[S(\underline{q})-\underline{t}]+(1-p)[S(\overline{q})-\overline{t}]$ | *(OBJ)* |
| s.t. | $\underline{t}-\underline{\theta}\underline{q}\geq 0$ | *(PC-E)* |
| | $\overline{t}-\overline{\theta}\overline{q}\geq 0$ | *(PC-I)* |
| | $\underline{t}-\underline{\theta}\underline{q}\geq \overline{t}-\underline{\theta}\overline{q}$ | *(IC-E)* |
| | $\overline{t}-\overline{\theta}\overline{q}\geq \underline{t}-\overline{\theta}\underline{q}$ | *(IC-I)* |
| | $\underline{q}_{\min}\leq \underline{q}\leq \underline{q}_{\max}$ | *(DC-E)* |
| | $\overline{q}_{\min}\leq \overline{q}\leq \overline{q}_{\max}$ | *(DC-I)* |
| | $\underline{t},\overline{t}\geq 0$ | |

where $\underline{q}_{\min},\overline{q}_{\min}\geq 0$. The *DC-E* and *DC-I* are the domain constraints of the efficient and inefficient collectors, respectively.

With complete information, the processor can tailor a different model for different collector types. In knowing the different contracts for each type, the processor realizes the upper and lower bounds on the net profit due to the fact that NRPA incorporates both types of collectors in a weighted manner. The complete information models can be written as:

**Efficient Contract Non-Regional Model**

$$
\begin{aligned}
\text{Maximize} \quad & S(\underline{q}) - \underline{t} \\
\text{s.t.} \quad & \underline{t} - \underline{\theta}\underline{q} \geq 0 \\
& \underline{q}_{\min} \leq \underline{q} \leq \underline{q}_{\max} \\
& \underline{t}, \underline{q} \geq 0
\end{aligned}
$$

**Inefficient Contract Non-Regional Model**

$$
\begin{aligned}
\text{Maximize} \quad & S(\bar{q}) - \bar{t} \\
\text{s.t.} \quad & \bar{t} - \bar{\theta}\bar{q} \geq 0 \\
& \bar{q}_{\min} \leq \bar{q} \leq \bar{q}_{\max} \\
& \bar{t}, \bar{q} \geq 0
\end{aligned}
$$

For the Regional Model, the principal offers different contracts to different regions. With different space and cost, the ease of collection of each region can vary greatly. Because each region carries the same assumption that the collectors can be typed as efficient or inefficient, it can be depicted with an example of $m$ independent regions as shown in Figure 5.2. Note that the Non-Regional Contract Model is a subset of the Regional Model.

Let $j$ be the index for regions, $j$ = 1, 2, …, $m$, and let $k$ be the index for distinct collector marginal cost values over all regions such that, $k$ = 1, 2, …, $m'$, where $m' \leq 2m$. Before modeling the problem with the MTPA model, it is necessary to perform a sorting algorithm. There are $m$ regions, hence up to $2m$ distinct marginal cost values. In ranking all $(\underline{\theta}_j, \bar{\theta}_j)$ values, $\forall j = 1, \ldots, m$, there are $m'$ number of distinct marginal cost values. A specific collector's marginal cost value is denoted as $\theta_{k,j}$.

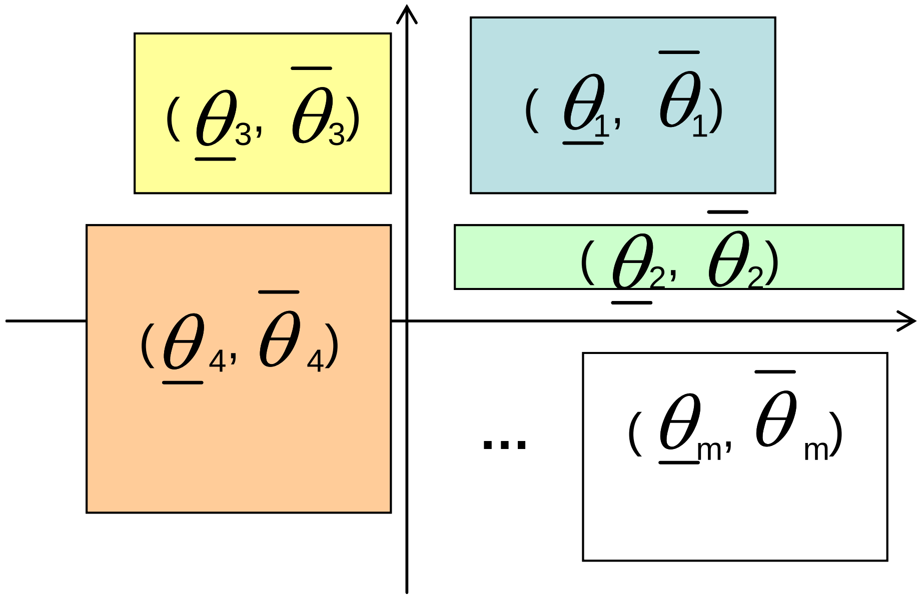



Figure 5.2: Regional Model Depiction

This ranking algorithm can be done in *O(mlogm)*, polynomial time in parameter *m*. The subscript $k = 1$ is the lowest marginal cost (most efficient type). The subscript $k = m'$, on the other hand, is the highest marginal cost (least efficient type). Moreover, $p_k$ denotes the probability of the collector having the $k^{th}$ lowest marginal cost, where $\sum_{k=1}^{m'} p_k = 1$.

With these notations and assumptions, the Regional Principal-Agent (RPA) Model can therefore be stated as:

**RPA Model**

| | | | |
|---|---|---|---|
| Maximize | $\sum_{k=1}^{m'} p_k[S(q_k) - t_k]$ | | *(OBJ)* |
| s.t. | $t_{m'} - \theta_{m',j} q_{m'} \geq 0$ | | *(PC-m')* |
| | $t_k - \theta_{k,j} q_k \geq t_{k+1} - \theta_{k,j} q_{k+1}$ | $k = 1, \dots, m'-1$ | *(IC-k)* |
| | $q_k \geq q_{k+1}$ | $k = 1, \dots, m'$-1 | *(MC-k)* |
| | $q_{\min k} \leq q_k \leq q_{\min k}$ | $k = 1, \dots, m'$ | *(DC-k)* |
| | $t_k, q_k \geq 0$ | $k = 1, \dots, m'$ | |

As explained in Laffont and Martimort (2002), the Revelation Principle ensures that it is possible to restrict the processor to offer simple menu contracts having as many options as the total number of types. When the processor captures all the types correctly, i.e., each collector is assigned a marginal cost from a list of all existing costs, the menu lump sum contract is optimal. As the processor offers a menu lump sum contract, the collector will truthfully select a contract alternative that corresponds to his or her type. The truthful direct revelation mechanism is denoted by $\{(t_1, q_1), (t_2, q_2), ..., (t_{m'}, q_{m'})\}$.

The RPA Model is solved to obtain the *m'* menu contract pairs. The next step is to convert the solution back to the original setting. The sorting of contract pairs can also be performed in *O(mlogm)*. For each region *j*, there are two values of *k*. For the smaller *k*, there is the contract of $(\underline{t}_j, \underline{q}_j)$. For the larger *k*, there is the contract of $(\bar{t}_j, \bar{q}_j)$. Eventually a contract is obtained for efficient and inefficient types for each region *j* in consideration.

Utilizing these contract models, the processor develops a menu of these lump sum contracts. The contract alternatives for both types of collectors become inputs for the strategic network model. This is the Sequential Model which is discussed in the subsequent section. I only consider the regional contract because of its generality in the following sections.

## 5.3 Sequential Model

The strategic network problem in the contract problem is essentially which collectors to offer contract alternatives to. Figure 5.3 summarizes the Sequential Model.

This is the decision in the collection stage of the RPS infrastructure. In the Sequential Model, the contract model is solved first. Then, the strategic network model is implemented. The following notations are employed in all variations of the models proposed in the following sections. Note that for the non-regional model, the index *j* is omitted.

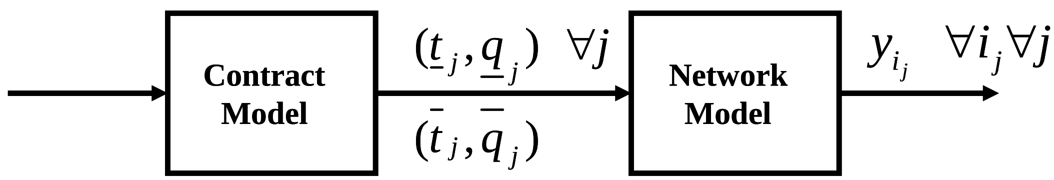



Figure 5.3: Sequential Model Summary

<u>Index</u>

$j$: index of regions, $j = 1, 2, \ldots, m$

$i_j$: index of collectors in region $j$, $i_j = 1, 2, \ldots, I_j$, where $n = \sum_j I_j$

$k$: index of the distinct set of collector marginal cost values, $k = 1, 2, \ldots, m'$, where $m' \le 2m$.

<u>Known Parameters</u>

$ct_j$: Transportation cost coefficient of region $j$, \$/mile.lb

$d_{i_j}$: Distance between processor and collector $i$ in region $j$, mile

$F_{i_j}$: Fixed cost to processor if collector $i$ in region $j$ joins the network, \$

$Q$: Target total material quantity required by the processor, lb

$B$: Material purchase budget limit for the processor, \$

$\pi_j$: Probability of a collector in region $j$ accepting the given contract

$p_j$: Probability of a collector in region $j$ being the efficient type

$\underline{\theta}_j$: Marginal cost of efficient collector in region $j$

$\bar{\theta}_j$ : Marginal cost of inefficient collector in region $j$

$\underline{t}_j$ : Lump sum transfer payment to efficient collector in region $j$, \$

$\bar{t}_j$ : Lump sum transfer payment to inefficient collector in region $j$, \$

$\underline{q}_j$ : Quantity material collected from efficient collector in region $j$, lb

$\bar{q}_j$ : Quantity material collected from inefficient collector in region $j$, lb

Decision Variables

$y_{i_j}$ : Binary variable for collector $i_j$

$$y_{i_j} = \begin{cases} 1 & \text{if contract is offered to collector } i \text{ in region } j \\ 0 & \text{otherwise} \end{cases}$$

With these notations, the strategic network model for the case of incomplete information can be stated as follows.

**Sequential Model (Incomplete Information)**

Maximize
$$S(\sum_j \pi_j \sum_{i_j} (p_j \underline{q}_j + (1-p_j)\bar{q}_j) y_{i_j}) - \sum_j \pi_j \sum_{i_j} (p_j \underline{t}_j + (1-p_j)\bar{t}_j) y_{i_j} \qquad \textit{(OBJ)}$$
$$-\sum_j \pi_j \sum_{i_j} ct_j d_{i_j} (p_j \underline{q}_j + (1-p_j)\bar{q}_j) y_{i_j} - \sum_j \pi_j \sum_{i_j} F_{i_j} y_{i_j}$$

s.t.
$$\sum_j \pi_j (\sum_{i_j} p_j \underline{q}_j y_{i_j}) + \sum_j \pi_j (\sum_{i_j} (1-p_j)\bar{q}_j y_{i_j}) \geq Q \qquad \textit{(QC)}$$
$$\sum_j \pi_j (\sum_{i_j} p_j \underline{t}_j y_{i_j}) + \sum_j \pi_j (\sum_{i_j} (1-p_j)\bar{t}_j y_{i_j}) \leq B \qquad \textit{(BC)}$$
$$y_{i_j} \in \{0,1\} \quad \forall i_j \forall j$$

The objective function (*OBJ*) represents the principal valuation collected product from all the chosen collectors in all regions minus the costs which include the lump sum transfer payments to all the chosen collectors in all regions, the transportation cost to move material from all the chosen collector sites in all regions, and the total fixed cost to the processor for including the chosen collectors in his or her network. The target quantity constraint (*QC*) requires that the total collected quantity from all the chosen collectors provide the required target quantity needed by the processor. The budget constraint (*BC*) suggests that the total of the lump sum transfer payments to the chosen collectors must be within the processor's material budget.

As mentioned in Chapter 1, one research goal is to understand the value of information. With complete information, the processor knows the type of each collector and wants to use this information to make sure s/he does not overpay for the product. In comparing the processor's net profits obtained from the complete information and incomplete information models, the processor can have a better idea of how much s/he should pay to get the information (if possible). An example showing how to calculate the value of information is discussed in Section 5.5.5.

I also want to consider an intermediate case, which unlike the complete information case where all collectors accept the given contracts, each collector can accept or reject the offered contract. While the case where the collectors cannot reject a contract is called "complete information without collector's choice," the intermediate case is denoted as "complete information with collector's choice." Let *E* represent the set of efficient collectors and *IE* represent the set of inefficient collectors. Let $i_j \in E$ be the collector *i* in region *j* that belongs to set *E*. Similarly, let $i_j \in IE$ be the collector *i* in region *j* that belongs to set *IE*. The strategic network model with the assumption of complete information and collector's choice can be written as:

**Sequential Model (Complete Information)**

Maximize
$$S(\sum_j \pi_j \sum_{i_j \in E} \underline{q}_j y_{i_j} + \sum_j \pi_j \sum_{i_j \in IE} \overline{q}_j y_{i_j}) - \sum_j \pi_j \sum_{i_j \in E} \underline{t}_j y_{i_j} - \sum_j \pi_j \sum_{i_j \in IE} \overline{t}_j y_{i_j} \qquad (OBJ)$$
$$-\sum_j \pi_j \sum_{i_j \in E} ct_j d_{i_j} \underline{q}_j y_{i_j} - \sum_j \pi_j \sum_{i_j \in IE} ct_j d_{i_j} \overline{q}_j y_{i_j}$$
$$-\sum_j \pi_j \sum_{i_j} F_{i_j} y_{i_j}$$

s.t.
$$\sum_j \pi_j \sum_{i_j \in E} \underline{q}_j y_{i_j} + \sum_j \pi_j \sum_{i_j \in IE} \overline{q}_j y_{i_j} \geq Q \qquad (QC)$$
$$\sum_j \pi_j \sum_{i_j \in E} \underline{t}_j y_{i_j} + \sum_j \pi_j \sum_{i_j \in IE} \overline{t}_j y_{i_j} \leq B \qquad (BC)$$
$$y_{i_j} \in \{0,1\} \quad \forall i_j \forall j$$

This model is different from the incomplete information model in that it does not incorporate the probability of a collector being of a particular type. The probability of a collector accepting the contract terms, $\pi_j$, may inflate the quantity and transfer payment values because the collector has the option of rejecting the contract. For the perfect complete information case where the collector must accept the contract and does not have a choice, $\pi_j$ is equal to one for every *j*.

The Sequential Model is a tool to help the processor decide which collector(s) to offer the contract. With the contract decisions as inputs, the model is an integer program (IP). The performance of the Sequential Model will be discussed in Section 5.5.

The Simultaneous Model where contract and strategic network decisions are solved at the same time is discussed in the next section.

## 5.4 Simultaneous Model

In the Simultaneous Model, both contract and network problems are solved at once. The anticipated advantages of the Simultaneous Model when compared to the Sequential Model are both the relative ease of implementation and the better net profit obtained by the processor. Figure 5.4 summarizes the Simultaneous Model.

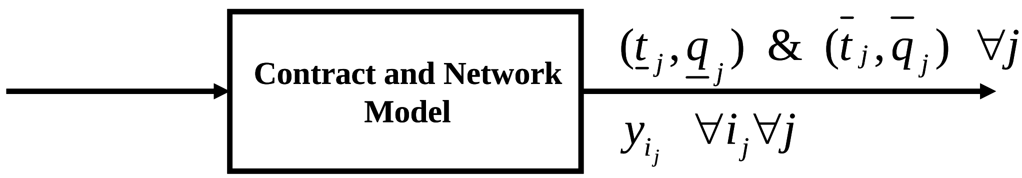



Figure 5.4: Simultaneous Model Summary

The simultaneous approach is expected to provide higher net profit to the processor because it maximizes the end objective by incorporating valuation, transfer payment, transportation cost, and fixed cost of the collectors simultaneously. This higher net profit expectation will be discussed in Section 5.5.

It is necessary to include an additional index and variable for the Simultaneous Model. Let *l* be the index of the unique rank order marginal costs such that *l* = 1, 2, …, *L*. The relation between *l* and *j* is $(\underline{\theta}_j, \bar{\theta}_j) = \{\theta_{\alpha(l_1)}, \theta_{\alpha(l_2)} \mid \alpha(l_1) = \alpha(l_2) = j, l_1 \le l_2\}$, $|l| = L \le J/2$. Also, let $q_l$ be the quantity material collected from the marginal cost rank order *l* collector, lb. The Simultaneous Model can be written as follows:

**Simultaneous Model (Incomplete Information)**

Maximize $$S(\sum_j \sum_{i_j} \pi_j (p_j \underline{q}_j + (1-p_j)\overline{q}_j) y_{i_j}) - \sum_j \sum_{i_j} \pi_j (p_j \underline{t}_j + (1-p_j)\overline{t}_j) y_{i_j} \qquad (OBJ)$$

$$-\sum_j \sum_{i_j} \pi_j ct_j d_{i_j} (p_j \underline{q}_j + (1-p_j)\overline{q}_j) y_{i_j} - \sum_j \sum_{i_j} \pi_j F_{i_j} y_{i_j}$$

s.t. $$\sum_j \sum_{i_j} \pi_j (p_j \underline{q}_j + (1-p_j)\overline{q}_j) y_{i_j} \geq Q \qquad (QC)$$

$$\sum_j \sum_{i_j} \pi_j (p_j \underline{t}_j + (1-p_j)\overline{t}_j) y_{i_j} \leq B \qquad (BC)$$

$$t_L - \theta_L q_L \geq 0 \qquad (PC)$$

$$t_l - \theta_l q_l \geq t_{l+1} - \theta_l q_{l+1} \qquad \forall l = 1,\ldots,L-1 \qquad (IC)$$

$$q_l \geq q_{l+1} \qquad \forall l = 1,\ldots,L-1 \qquad (MC)$$

$$q_l \geq q_{l\min} \qquad \forall l \qquad (LQC)$$

$$q_l \leq q_{l\max} \qquad \forall l \qquad (UQC)$$

$$\underline{t}_j, \overline{t}_j, t_l \geq 0 \qquad \underline{q}_j, \overline{q}_j, q_l \geq 0$$

$$y_{i_j} \in \{0,1\} \quad \forall i_j \forall j$$

The *OBJ* function includes terms to represent the material valuation, transfer payment costs, transportation costs, and fixed costs. The *PC*, *IC*, *MC* constraints are from the MTPA contract model. The *LQC* and *UQC* constraints enforce the quantity domain. The remaining constraints follow explanations from the previous sections.

I expect that the Simultaneous Model will have higher net profit than the Sequential Model because of the single solution step. A numerical study is performed in the following section. Insights of the Sequential and Simultaneous models are given, which confirm the expectation.

## 5.5 Numerical Study

In assessing the Sequential and Simultaneous Models, I performed numerical studies on many small examples, though only four examples are reported in detail. In the final example, I compute the value of information for a small instance via the Complete and Incomplete Information Simultaneous Models. The expected value on information is found from combinations of collectors being efficient or inefficient. For simplicity, these are examples of a non-regional problem. At the end of this section, the advantages and disadvantages of both models are discussed.

### 5.5.1 Example 1

In this example, I assume that there are three available collectors. The non-regional problem has marginal costs of 0.05 and 0.08 \$/lb for efficient and inefficient collectors, respectively. The probability of being an efficient collector, $p$, has a value of 0.5. The processor has a coefficient of valuation of 10 with a target quantity and budget of 150 lb and \$20, respectively. The collectors have distances from the processor and fixed costs of 100 mile and \$10, 150 miles and \$5, and 50 miles and \$15, respectively. Finally, the operating quantity of an efficient collector is between 20 and 300 lb. On the other hand, the inefficient collector has an operating quantity between 10 and 100 lb.

The Sequential and Simultaneous Model solutions are obtained using the BARON (2007) commercial solver on the Window XP computer platform. Table 5.1 reports the solutions. Each row represents solutions for different values of the probability of acceptance, $\pi$.

Table 5.1: Solutions to Example 1

| $\pi$ | Sequential Model<br>Eff. Contract ($,lb) | Ineff. Contract ($,lb) | $(y_1,y_2,y_3)$ | Net Profit $ | Simultaneous Model<br>Eff. Contract ($,lb) | Ineff. Contract ($,lb) | $(y_1,y_2,y_3)$ | Net Profit $ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | (18, 300) | (8, 100) | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible |
| 0.1 | (18, 300) | (8, 100) | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible |
| 0.2 | (18, 300) | (8, 100) | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible |
| 0.5 | (18, 300) | (8, 100) | (1,1,1) | 116.2 | (18, 300) | (8, 100) | (1,1,1) | 116.2 |
| 0.8 | (18, 300) | (8, 100) | (1,0,0) | 96.1 | (18, 300) | (7, 91) | (1,0,0) | 116.9 |
| 0.9 | (18, 300) | (8, 100) | (1,0,0) | 100.9 | (17, 300) | (5, 66) | (1,0,0) | 116.8 |
| 1.0 | (18, 300) | (8, 100) | (1,0,0) | 103.4 | (16, 300) | (4, 46) | (1,0,0) | 116.5 |

For the extremely small $\pi$ values, both the Sequential and Simultaneous Models give infeasible solutions. For the median $\pi$ value, both models result in the same solution. However, there are obvious differences between the two models for relatively large $\pi$ values. The contract for an inefficient collector varies greatly. This example suggests that the Simultaneous Model may yield better net profits for all values of $\pi$.

To more clearly see the difference in solution feasibility for the two models, Example 2 is introduced.

### 5.5.2 Example 2

This example has many of the same parameters as Example 1. The differences are 1) a target quantity of 300 lb, 2) a budget of $30, and 3) an operating quantity between 100 and 3000 lb for both types of collectors. Table 5.2 summarizes the solutions.

Table 5.2: Solutions to Example 2

| $\pi$ | Sequential Model<br>Eff. Contract ($,lb) | Ineff. Contract ($,lb) | $(y_1,y_2,y_3)$ | Net Profit $ | Simultaneous Model<br>Eff. Contract ($,lb) | Ineff. Contract ($,lb) | $(y_1,y_2,y_3)$ | Net Profit $ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | (212, 3000) | (165, 2066) | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible |
| 0.1 | (212, 3000) | (165, 2066) | (0,0,1) | 130.1 | (205, 3000) | (146, 1818) | (1,0,1) | 158.0 |
| 0.2 | (212, 3000) | (165, 2066) | Infeasible | Infeasible | (205, 3000) | (146, 1818) | (0,0,1) | 163.5 |
| 0.5 | (212, 3000) | (165, 2066) | Infeasible | Infeasible | (8, 100) | (132, 2580) | (0,0,1) | 168.6 |
| 0.8 | (212, 3000) | (165, 2066) | Infeasible | Infeasible | (8, 100) | (80, 1530) | (0,0,1) | 163.4 |
| 0.9 | (212, 3000) | (165, 2066) | Infeasible | Infeasible | (8, 100) | (70, 1336) | (0,0,1) | 161.7 |
| 1.0 | (212, 3000) | (165, 2066) | Infeasible | Infeasible | (8, 100) | (62, 1180) | (0,0,1) | 160.0 |

The Sequential Model yields infeasibility for most $\pi$ values, while the Simultaneous Model mostly finds feasible solutions. Moreover, for the $\pi$ value of 0.1 with feasible solutions for both models, there are significant differences in contracts. At the same time, the Sequential Model only offers a contract to Collector 3 whereas the Simultaneous offers a contract to Collectors 1 and 3.

Varying the marginal costs such that the efficient and inefficient collectors differ greatly is addressed in Example 3.

### 5.5.3 Example 3

Consider an example with three available collectors. The non-regional problem has marginal costs of 0.02 and 0.10 $/lb for efficient and inefficient collectors, respectively. As a result, the efficient collector has a marginal cost 5 times cheaper than that of inefficient collectors. The processor has a target quantity and budget of 300 lb and $30, respectively. The operating quantity for all collectors is between 10 and 5000 lb. The remaining parameters hold the same values as Examples 1 and 2. Table 5.3 reports the solutions.

The solutions are similar to Example 2. The Sequential Model remains infeasible for most $\pi$ values. Moreover, the contracts for efficient and inefficient types differ greatly between models, especially for inefficient collectors. For $\pi$ values of 0.1 and 0.2 which are feasible solutions for both models, the collection network has a much different structure. The Simultaneous Model suggests that the processor should offer contracts to all collectors. The Sequential Model, on the other hand, wants to select only one or two collectors. Again, for these two $\pi$ values, the processor net profits for the Simultaneous Model are relatively greater.

Table 5.3: Solutions to Example 3

| | Sequential Model | | | | Simultaneous Model | | | |
|---|---|---|---|---|---|---|---|---|
| $\pi$ | Eff. Contract ($,lb) | Ineff. Contract ($,lb) | $(y_1,y_2,y_3)$ | Net Profit $ | Eff. Contract ($,lb) | Ineff. Contract ($,lb) | $(y_1,y_2,y_3)$ | Net Profit $ |
| 0.0 | (162, 5000) | (77, 772) | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible |
| 0.1 | (162, 5000) | (77, 772) | (1,0,1) | 183.8 | (100.8, 5000) | (1, 10) | (1,1,1) | 203.3 |
| 0.2 | (162, 5000) | (77, 772) | (0,0,1) | 190.5 | (99, 4910) | (1, 10) | (1,1,1) | 245.0 |
| 0.5 | (162, 5000) | (77, 772) | Infeasible | Infeasible | (101, 5000) | (1, 10) | (0,0,1) | 264.7 |
| 0.8 | (162, 5000) | (77, 772) | Infeasible | Infeasible | (74, 3660)) | (1, 10) | (0,0,1) | 275.2 |
| 0.9 | (162, 5000) | (77, 772) | Infeasible | Infeasible | (66, 3243) | (1, 10) | (0,0,1) | 273.4 |
| 1.0 | (162, 5000) | (77, 772) | Infeasible | Infeasible | (59, 2910) | (1, 10) | (0,0,1) | 271.6 |

To better understand the difference between the models, a problem with more collectors and high marginal costs is examined in Example 4.

### 5.5.4 Example 4

In this example, the efficient and inefficient collectors have marginal costs of 10 and 12 $/lb, respectively. The processor has a target quantity of 25 lb and a budget of $1000. (imagine this example as the recycling of a high-value low-volume product). The operating quantity domain for every collector is 1 to 30 lb. There are five available collectors with distance from processor and fixed cost of 100 miles and $10, 150 miles and $5, 50 miles and $15, 60 miles and $12, and 120 miles and $8, respectively.

In this example, the Sequential Model cannot find a feasible solution for all $\pi$ values because of the very low value of contract solutions. On the other hand, the Simultaneous Model performs reasonably well for $\pi$ value of 0.2 or more.

Table 5.4: Solutions to Example 4

| | Sequential Model | | | | Simultaneous Model | | | |
|---|---|---|---|---|---|---|---|---|
| $\pi$ | Eff. Contract ($,lb) | Ineff. Contract ($,lb) | $(y_1,y_2,y_3,y_4,y_5)$ | Net Profit $ | Eff. Contract ($,lb) | Ineff. Contract ($,lb) | $(y_1,y_2,y_3,y_4,y_5)$ | Net Profit $ |
| 0.0 | (12, 1) | (12, 1) | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible |
| 0.1 | (12, 1) | (12, 1) | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible | Infeasible |
| 0.2 | (12, 1) | (12, 1) | Infeasible | Infeasible | (300, 25) | (300, 25) | (1,1,1,1,1) | 138.7 |
| 0.5 | (12, 1) | (12, 1) | Infeasible | Infeasible | (300, 25) | (300, 25) | (0,1,0,0,1) | 141.8 |
| 0.8 | (12, 1) | (12, 1) | Infeasible | Infeasible | (125, 10.4) | (125, 10.4) | (1,0,0,1,1) | 124.7 |
| 0.9 | (12, 1) | (12, 1) | Infeasible | Infeasible | (333, 28) | (333, 28) | (0,1,0,0,0) | 143.6 |
| 1.0 | (12, 1) | (12, 1) | Infeasible | Infeasible | (300, 25) | (300, 25) | (0,1,0,0,0) | 143.1 |

### 5.5.5 Example 5

In this example, three collectors are considered. The efficient and inefficient collectors have marginal costs of 0.1 and 0.3 \$/lb, respectively. The processor has a target quantity of 150 lb and a budget of \$50. The operating quantity domain for every collector is 10 to 200 lb. Collectors 1, 2, and 3 have distance from processor and fixed cost of 50 miles and \$150, 150 miles and \$50, and 100 miles and \$100, respectively. The coefficient of transportation has value of 0.005 \$/lb-mile. These collectors have the common probability of being efficient and probability of accepting a contract of 0.5 and 0.8, respectively. Finally, the processor has a linear coefficient of valuation of 5 \$/lb.

Using the Simultaneous Model in the complete and incomplete information cases, the contracts, offering decisions, and processor's net profits are displayed in Table 5.5. The solutions to the complete information without collector's choice and with collector's choice are presented. With three collectors, there are a total of eight combinations for collector's efficiency. Let *e* represent efficient type and *ie* represent inefficient type.

From the Simultaneous Model with incomplete information, the processor should offer a contract menu for all three collectors, yielding the net profit of \$1005. If the processor identifies Collectors 1 and 2 are efficient and Collector 3 is inefficient, then the value of information for the complete information case without collector's choice is \$555 (\$1560 - \$1005). In obtaining collector's information (if possible), the processor should not pay more than \$555. Similarly for the complete information case with collector's choice, the value of information is calculated from the difference in processor's net profits. It has the value of information of \$415.

Table 5.5: Solutions to Example 5

| Incomplete Information | | | |
|---|---|---|---|
| Eff. Contract ($,lb) | Ineff. Contract ($,lb) | $(y_1, y_2, y_3)$ | Net Profit $ |
| (29, 200) | (13, 43) | (1, 1, 1) | 1005 |

| | Complete Information - Without Collector's Choice | | | | |
|---|---|---|---|---|---|
| Combination (Collector1,Collector2,Collector3) | Collector 1 ($,lb) | Collector 2 ($,lb) | Collector 3 ($,lb) | $(y_1, y_2, y_3)$ | Net Profit $ |
| (e, e, e) | (20, 200) | (10, 100) | (20, 200) | (1, 1, 1) | 1925 |
| (e, e, ie) | (20, 200) | (20, 200) | (3, 10) | (1, 1, 0) | 1560 |
| (e, ie, e) | (20, 200) | (60, 200) | (20, 200) | (1, 0, 1) | 1560 |
| (e, ie, ie) | (20, 200) | (30, 100) | (6, 20) | (1, 1, 0) | 1125 |
| (ie, e, e) | (60, 200) | (20, 200) | (20, 200) | (0, 1, 1) | 1560 |
| (ie, e, ie) | (13, 43) | (20, 200) | (30, 100) | (0, 1, 1) | 1100 |
| (ie, ie, e) | (9, 31) | (30, 100) | (20, 200) | (0, 1, 1) | 1125 |
| (ie, ie, ie) | (26, 88) | (50, 167) | (25, 85) | (0, 1, 1) | 608 |

| | Complete Information - With Collector's Choice | | | | |
|---|---|---|---|---|---|
| Combination (Collector1,Collector2,Collector3) | Collector 1 ($,lb) | Collector 2 ($,lb) | Collector 3 ($,lb) | $(y_1, y_2, y_3)$ | Net Profit $ |
| (e, e, e) | (20, 200) | (20, 200) | (20, 200) | (1, 1, 1) | 1872 |
| (e, e, ie) | (20, 200) | (20, 200) | (23, 75) | (1, 1, 1) | 1420 |
| (e, ie, e) | (20, 200) | (22.5, 75) | (20, 200) | (1, 1, 1) | 1445 |
| (e, ie, ie) | (20, 200) | (43, 142) | (7, 22) | (1, 1, 0) | 1032 |
| (ie, e, e) | (23, 75) | (20, 200) | (20, 200) | (1, 1, 1) | 1395 |
| (ie, e, ie) | (13, 43) | (20, 200) | (43, 142) | (0, 1, 1) | 1020 |
| (ie, ie, e) | (13, 43) | (43, 142) | (20, 200) | (0, 1, 1) | 1032 |
| (ie, ie, ie) | (13, 43) | (60, 200) | (60, 200) | (0, 1, 0) | 592 |

If the processor is uncertain whether each collector is efficient or not, s/he can compute the expected value of information (EVOI), calculating with the distribution of *p*. With each collector having a probability of being efficient of 0.5, each combination has a probability of 0.125 ($0.5^3$). The EVOI without collector's choice is $315, which is the expected net profit of $1320 minus the incomplete information net profit of $1005. Similarly, the EVOI with collector's choice is calculated from the expected net profit of $1226 minus $1005 to be $221. Knowing the EVOI, the processor recognizes the average benefit from knowing collector's information. The cost in obtaining the private information should not exceed the EVOI.

The insights from the above examples (and other runs performed) are reported in the next section.

### 5.5.6 Insights

From the above examples, one can clearly observe that 1) the Simultaneous Model seems to have more feasibility solutions for given probability of acceptance, $\pi$ and 2) the Simultaneous Model yields better net profit for given $\pi$ when both models are feasible.

To explain these results, it is necessary to inspect the Sequential and Simultaneous Models. The Simultaneous Model has an objective function which includes contract, transportation, and fixed cost terms. It has network constraints (*QC* and *BC*), contract constraints (*PC*, *IC*, *MC*, and *DC*), and variables constraints. On the other hand, the Sequential Model consists of two sub-models, which are the MTPA Contract Model and the Strategic Network Model. The MTPA Contract Model has objective function with only contract terms. It has *PC*, *IC*, *MC*, *DC*, and variables constraints. The Strategic Network Model has an objective function with contract, transportation, and fixed cost terms. These are identical to the objective function of the Simultaneous Model. The Strategic network has only *QC*, *BC*, and variables constraints. I exploit Figure 5.5 in explaining these results.

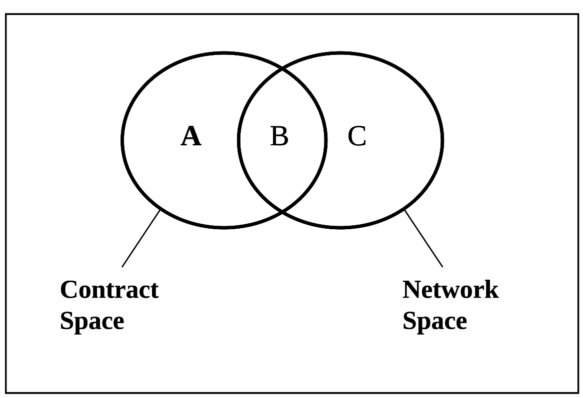



Figure 5.5: Feasibility Region of Simultaneous and Sequential Models

With an identical objective function in the final decisions for both models, concentration is on the feasibility region. In Figure 5.5, the left circle represents feasible contract solution space ($q$ and $t$ continuous variables). The right circle displays feasible strategic network solution space (binary variables). The B area contains the feasible solutions to the overall problem. The Simultaneous Model makes sure that the solution (if any) comes from this B area. However, the Sequential Model solves the MTPA contract model to find a possible solution (if any) to be in either A or B areas. If the solution is in A, infeasibility is immediately found in the strategic network model. However, if the solution is in B, a feasible solution is obtainable. Moreover, the solution to the MTPA contract is not necessarily the same with the contract solution from the Simultaneous Model due to the differences in the objective function. The MTPA contract (Sequential Model) model's objective function does not incorporate transportation and fixed cost in solving for the solution. As a result, the solution could be the best in terms of the contract offered but does not include information on transportation and fixed costs at all, which causes a discrepancy in solving for the solution in the strategic model.

The above explanations suggest that the Simultaneous Model will always dominate the Sequential Model. Moreover, it is easier to implement. Although the Simultaneous Model is MINLP, it is not much harder than solving NLP and IP models consecutively in the Sequential Model. With superior solution quality, the Simultaneous Model is recommended over the Sequential Model for practitioners, supporting the hypothesis that co-designing collection network and contracts for materials yield a better net processor's profit than designing each component individually and then combining those decisions. From this point on in the research, the Simultaneous Model is utilized for solving lump sum contract and network problems.

An additional model to solve the lump sum contract and network problem and implementing the concept of stochastic programming will be proposed and discussed in the following chapter. This addresses the limitation in the model created by the representation of the acceptance of the contract by the collector. In reality this would be a two-stage process in which the contracts are offered and then infrastructure put in place on the basis of which collectors are accepted.

# CHAPTER 6

# STOCHASTIC PROGRAMMING LUMP SUM MODEL

In this chapter a Stochastic Programming (SP) Model for determining lump sum contracts is presented. In determining the lump sum contracts, the SP approach is quite different from the deterministic programming (DP) approach described in Chapter 5 in that it considers all possible outcome scenarios. In the SP approach, there are solutions for each possible scenario (recourse). The SP Model determines regions to operate, contracts for each region, hub locations for each region, and collected material allocations for each site.

The chapter begins with an overview of SP methodology in Section 6.1 followed by a problem description in Section 6.2. The SP model is presented in Section 6.3 with a description of parameters, decision variables, assumptions, and the MINLP Model. Finally, a numerical study is performed in Section 6.4.

## 6.1 Stochastic Programming Overview

Stochastic programming (SP) is a class of optimization problems which considers uncertainty. The major assumptions of SP require a finite number of stages and exogenous uncertainties. SP is typically used when dealing with evolving data and making decisions without complete data. In this situation, one may want to develop models whose plans are evaluated against different scenarios to represent alternative outcomes.

In this thesis the specific SP formulation considered is called a *two-stage problem with recourse*. The recourse program has some decisions or recourse actions which can be taken after the uncertainty (or the random experiment) is resolved. The set of decisions can be divided into two groups which are first-stage and second-stage decisions. The former decisions are taken before the experiment in the first period. On the other hand, the latter decisions are taken after the experiment in the second stage.

In presenting a generic SP Model, the notation of Birge and Louveaux (1997) is followed. Let $\omega$ represent the random event. The first stage decisions are represented by vector *x*, while second-stage decisions are represented by vector *y* or $y(\omega)$ or $y(\omega, x)$. The random vector, $\xi(\omega)$, is only known after the random experiment. The generic SP Model can be written as:

$$
\begin{aligned}
\min \quad & c^T x + E_{\xi}[\min Q(x, \xi(\omega))] \\
s.t. \quad & Ax = b \\
& x \geq 0 \\
Q(x, \xi(\omega)) &= \min_y \{q(\omega)^T y \mid Wy = h(\omega) - T(\omega)x, y \geq 0\}
\end{aligned}
$$

The objective function consists of a deterministic term ($c^T x$) and an expectation of the second-stage objectives ($q(\omega)^T y$) taken over the entire random event set ($\omega$). The constraints include 1) first-stage only ($Ax = b$), 2) second-stage dependent on the first stage ($Wy = h(\omega) - T(\omega)x$), and 3) nonnegative constraints ($x \geq 0$ and $y \geq 0$).

As mentioned in Chapter 2, SP Models are widely studied in a variety of applications. A SP Model for the contract and strategic network problem is presented in the following section.

## 6.2 Model Description

The contract and strategic network problem can be modeled as a two-stage problem. In the first stage, the processor determines 1) regions to operate, 2) contracts for each region, and 3) collection hub(s) for each region. Then s/he offers contracts to the individual collection sites. The uncertainty is whether each collection site accepts or rejects the given contracts. In the second stage, the processor determines the assignment of collections to hubs and the vehicle routing. The SP stages and decisions are summarized in Figure 6.1.

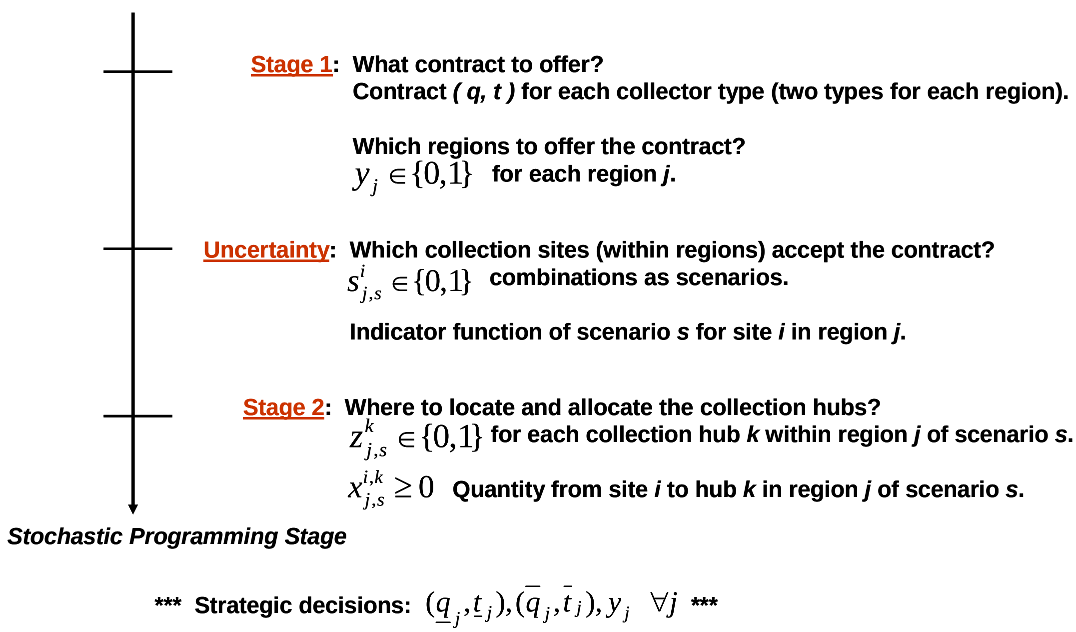



Figure 6.1: Contract and Network Design Stochastic Programming Model

The problem is depicted, step by step, in Figure 6.2. In the first stage, the processor selects a subset of regions to offer a contract to ($y_j = 1$) [see Figure 6.2 a)]. Figure 6.2 b) displays the scenarios scheme. Each collection site can accept or reject the contract, independently of other collection sites. The combinations of acceptances and rejections constitute the scenarios. The second stage decisions of a particular region are described in Figure 6.2 c). For each scenario, the acceptance decisions of collection sites are known. Hence, the processor makes decisions on hub(s) selection and collected material allocations for each hub.

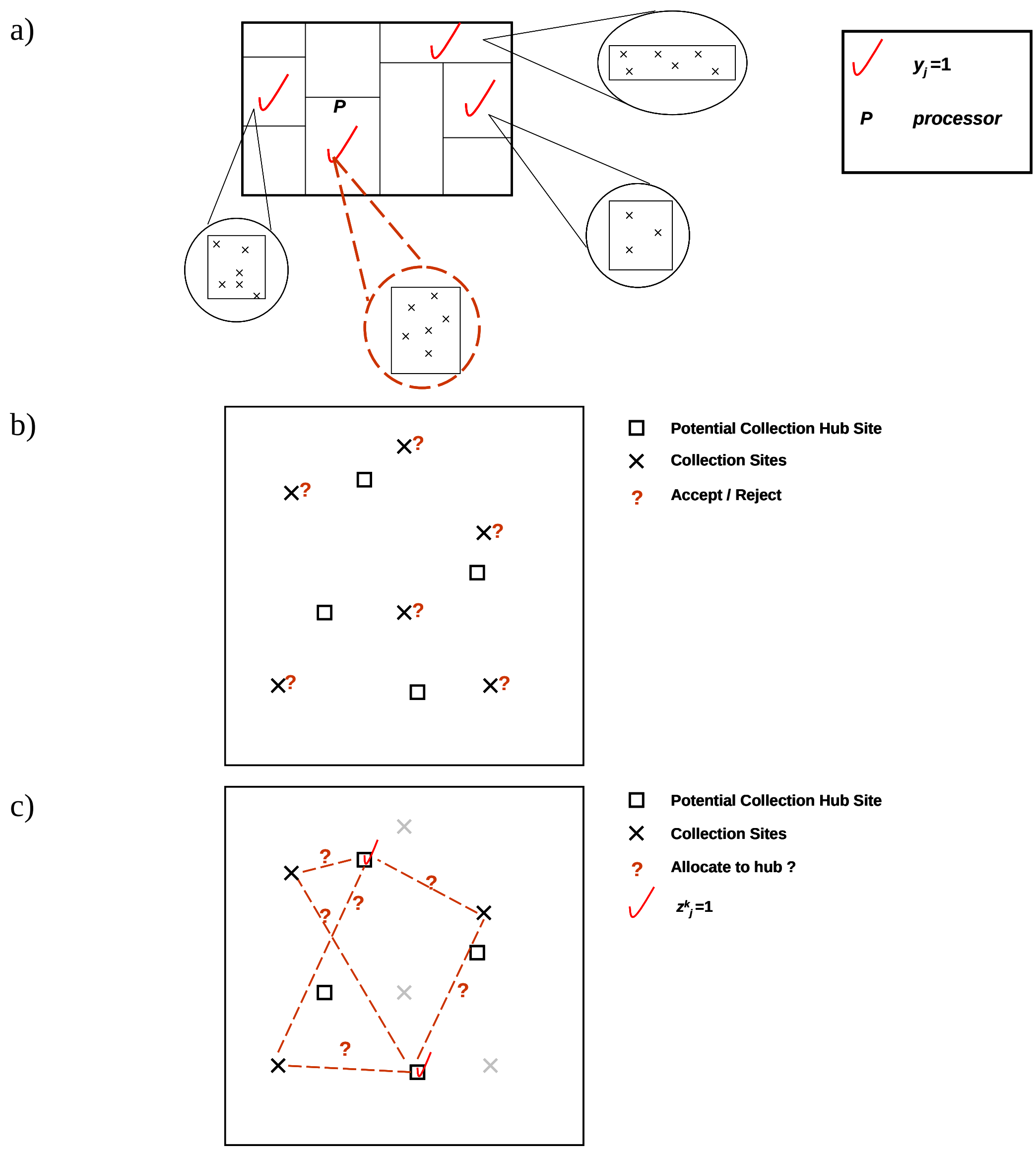



Figure 6.2: Depiction of Contract and Strategic Network Design SP Model- a) Stage 1 decisions, b) Stage 2 scenarios, and c) Stage 2 decisions

The optimization model, generally described in Figure 6.3, maximizes net profit which is comprised of the first stage revenue and costs and the second stage costs. The relevant constraints can be broken into three groups. The first group includes first stage constraints which do not involve second stage decisions. The second group includes the constraints linking the first stage decisions with the second stage decisions. The last group contains only the constraints affecting the second stage decisions.

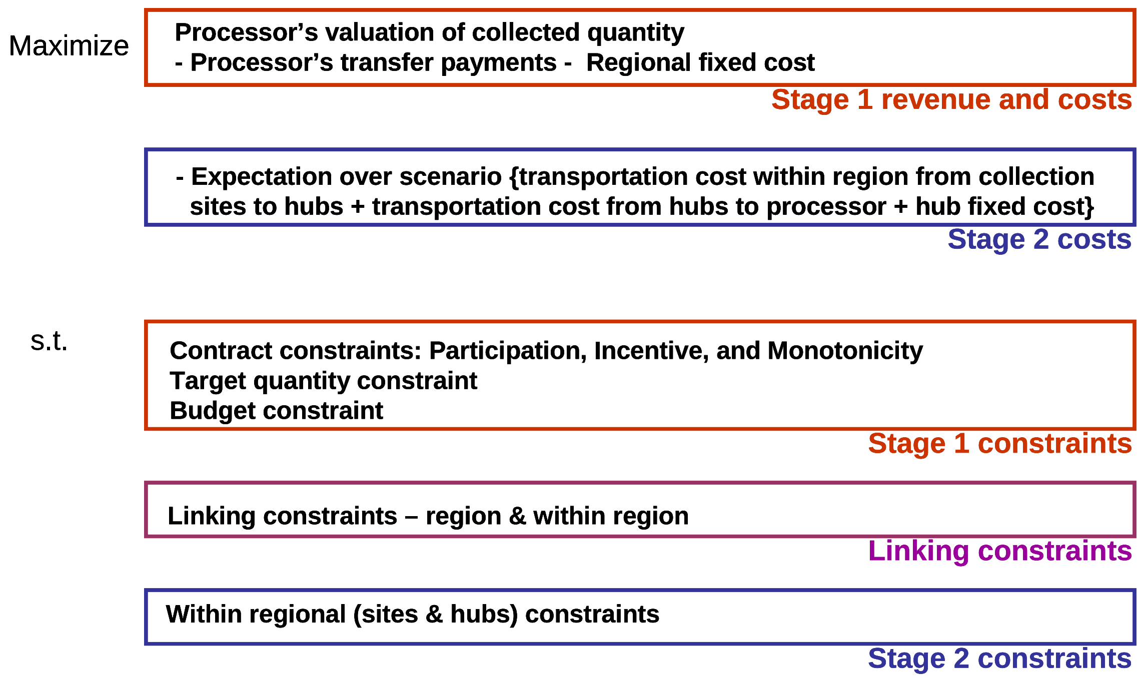



Figure 6.3: Stochastic Programming Model Description

The second stage costs and constraints can further be decomposed into two parts, see Figure 6.4. The first part includes decisions associated with collection sites and collection hubs. The objective function costs associated with this part include transportation between collection sites and collection hubs and the hub site fixed costs. I assume that the transportation mode is truck and move quantities are either truckload

(TL) or less than truckload (LTL). The decisions associated with the second part focus on transportation between collection hubs and processor. For this chapter, I assume that the mode of transportation can be either truck (TL) or rail, depending on the region.

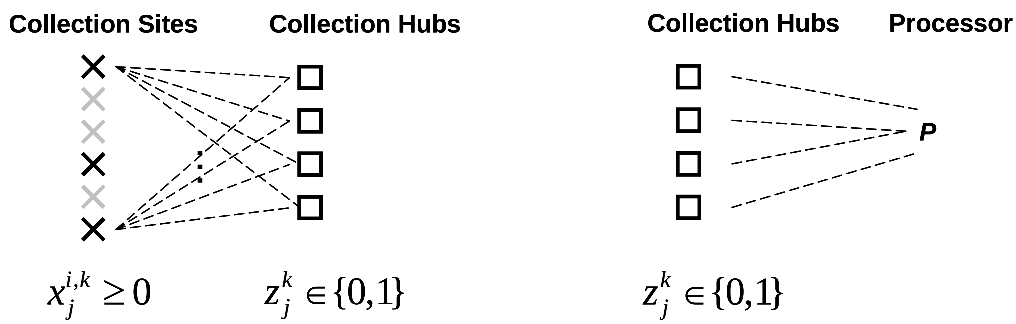



Figure 6.4: Decomposition of Stage 2 Costs and Constraints

Using the model description of the contract and strategic network design, the SP Model is next presented in the following section.

## 6.3 Stochastic Programming Lump Sum Contract Model

Before starting the model, it is necessary to describe the model indices and parameters. The many indices and parameters previously implemented in Chapters 3 and 5 are, for the most part, consistent with this notation.

The indices are:

$j$ : index of the region, $j = 1, 2, \ldots, J$

$i_j$ : index of collectors in region $j$, $i_j = 1, 2, \ldots, I_j$

$k_j$ : index of hubs in region $j$, $k_j = 1, 2, \ldots, K_j$

$l$ : index of unique ranking order of collector marginal costs, $l = 1, 2, \ldots, L$

$s$ : index of scenario, $s = 1, 2, \ldots, S$

One can see that the quantity and transfer payment implements two indices, *j* and *l*. The relation is; $Let\ (\underline{\theta}_j, \overline{\theta}_j) = \{\theta_{\alpha(l_1)}, \theta_{\alpha(l_2)} \mid \alpha(l_1) = \alpha(l_2) = j, l_1 \le l_2\}$, $|l| = L \le J/2$.

The known parameters include:

$c_{truck\ j}$ : LTL transportation cost coefficient of region *j*, \$/mile.lb

$c_j^k$ : TL transportation cost coefficient from hub *k* to processor of region *j*, \$/mile.tl

$kv$ : Coefficient of valuation of processor, \$/lb

$d_j^{i,k}$ : Distance from collection site *i* to hub *k* of region *j*, mile

$d_j^k$ : Distance from hub *k* to processor of region *j*, mile

$F_j$ : Fixed cost to operate in region *j*, \$

$F_j^k$ : Fixed cost to open hub *k* in region *j*, \$

$Q$ : Target total material quantity required by the processor, lb

$B$ : Material budget of the processor, \$

$p_s(s)$ : Probability associated with occurrence of scenario *s*

$\pi_j$ : Probability of a collector in region *j* accepting the given contract

$p_j$ : Probability of a collector in region *j* being the efficient type

$\underline{\theta}_j$ : Marginal cost of efficient collector in region *j*, \$/lb

$\overline{\theta}_j$ : Marginal cost of inefficient collector in region *j*, \$/lb

$\theta_l$ : Marginal cost of rank order *l*, \$/lb

The decision variables include:

$\underline{t}_j$ : Lump sum transfer payment to efficient collector in region *j*, \$

$\overline{t}_j$ : Lump sum transfer payment to inefficient collector in region *j*, \$

$t_l$ : Lump sum transfer payment to the rank order *l* collector, \$

$\underline{q}_j$ : Quantity material collected from efficient collector in region *j*, lb

$\overline{q}_j$ : Quantity material collected from inefficient collector in region *j*, lb

$q_l$ : Quantity material collected from the rank order *l* collector, lb

$y_j$ : Binary variable of region *j*

$$y_j = \begin{cases} 1 & \text{if contract is offered to region } j \\ 0 & \text{otherwise} \end{cases}$$

$z_{j,s}^{k}$ : Binary variable of hub *k*

$$z_{j,s}^{k} = \begin{cases} 1 & \text{if hub } k \text{ is selected in region } j \text{ of scenario } s \\ 0 & \text{otherwise} \end{cases}$$

$x_{j,s}^{i,k}$ : Quantity material shipped from site *i* to hub *k* in region *j* of scenario *s*, lb

There are two indexing systems for the SP Lump Sum Model. While one has a regional index, the other has a ranking order index. Figure 6.5 demonstrates the relation of the indices *j* and *l*.

$(\underline{\theta}_j, \overline{\theta}_j)$ (\$/lb): $(\underline{\theta}_1, \overline{\theta}_1) = (0.1, 0.5), (\underline{\theta}_2, \overline{\theta}_2) = (0.2, 0.4), (\underline{\theta}_3, \overline{\theta}_3) = (0.8, 1.2)$

| Rank Order | 0.1 | 0.2 | 0.4 | 0.5 | 0.8 | 1.2 |
|---|---|---|---|---|---|---|
| j | 1 | 2 | 2 | 1 | 3 | 3 |
| l | 1 | 2 | 3 | 4 | 5 | 6 |
| $\alpha : l \rightarrow j$ | 1 | 2 | 2 | 1 | 3 | 3 |

Figure 6.5: Example Showing the Relation between *j* and *l*

The value of $p_s(s)$ is obtained directly from the product of the corresponding $\pi_j$ and $p_j$. This follows the independence assumption for collection sites stated in Chapter 1. As an example of a four-collection site problem, where Sites 1 and 2 belong to Region 1 and Sites 3 and 4 belong to Region 2. The probability of the scenario with Site 1 rejecting a contract, Site 2 accepting a contract while being efficient, Site 3 accepting a contract while being inefficient, and Site 4 rejecting a contract is $(1-\pi_1)(\pi_1 p_1)(\pi_1(1-p_2))(1-\pi_2)$. The probability of each scenario can be computed in a similar manner.

Again repeating that the major assumptions still hold, there are two additional assumptions for the SP Model. The first assumption is that the hub construction is instantaneous, i.e., time is not an issue. The second assumption is that there is no cross-regional transportation, i.e., the processor cannot utilize hubs and sites to collect from other regions.

Having described the indices, parameters, decision variables, and assumptions, the SP Model can now be presented. The SP Lump Sum Model is posed as follows:

**Stochastic Programming Lump Sum Model**

Maximize

$$kv\sum_{j}^{J}\pi_j(p_j\underline{q}_j+(1-p_j)\overline{q}_j)I_j y_j-\sum_{j}^{J}\pi_j(p_j\underline{t}_j+(1-p_j)\overline{t}_j)I_j y_j-\sum_{j}^{J}F_j y_j \quad \text{(OBJ)}$$

$$-\sum_{s}^{S}p_s(s)[\sum_{j}^{J}[\sum_{i_j}^{I_j}\sum_{k_j}^{K_j}c_{truck\,j}d_j^{i,k}x_{j,s}^{i,k}+\sum_{k_j}^{K_j}\frac{\sum_{i_j}^{I_j}x_{j,s}^{i,k}}{TLC}c_j^k\,d_j^k+\sum_{k_j}^{K_j}F_j^k z_{j,s}^k]]$$

s.t.

$$\sum_{j}^{J}\pi_j(p_j\underline{q}_j+(1-p_j)\overline{q}_j)I_j y_j\geq Q \quad \text{(QC)}$$

$$\sum_{j}^{J}\pi_j(p_j\underline{t}_j+(1-p_j)\overline{t}_j)I_j y_j\leq B \quad \text{(BC)}$$

$$t_L-\theta_L q_L\geq 0 \quad \text{(PC)}$$

$$t_l-\theta_l q_l\geq t_{l+1}-\theta_l q_{l+1} \qquad \forall l=1,...,L-1 \quad \text{(IC)}$$

$$q_l\geq q_{l+1} \qquad \forall l=1,...,L-1 \quad \text{(MC)}$$

$$q_l\geq q_{l\min} \qquad \forall l \quad \text{(LQC)}$$

$$q_l\leq q_{l\max} \qquad \forall l \quad \text{(UQC)}$$

$$\sum_{i_j}^{I_j} x_{j,s}^{i,k} \le C^k z_{j,s}^k \qquad \forall k_j \forall j \forall s \qquad (HC)$$

$$\sum_{k_j}^{K_j} x_{j,s}^{i,k} = \underline{q}_j \qquad \forall j \quad \forall s \forall i \in \text{EfficientType\&AcceptContract} \qquad (AEC)$$

$$\sum_{k_j}^{K_j} x_{j,s}^{i,k} = \overline{q}_j \qquad \forall j \quad \forall s \forall i \in \text{InefficientType\&AcceptContract} \qquad (AIC)$$

$$\sum_{k_j}^{K_j} z_{j,s}^{k} \le K_j y_j \qquad \forall j \forall s \qquad (LC)$$

$$y_i \in \{0,1\} \quad \forall i, \quad z_{j,s}^k \in \{0,1\} \qquad \forall j \forall k \forall s$$

$$x_{j,s}^{i,k} \ge 0 \qquad \forall i \forall k \forall j \forall s$$

$$\underline{q}_j, \overline{q}_j, \underline{t}_j, \overline{t}_j \ge 0 \qquad \forall j$$

The objective function (*OBJ*) maximizes the processor's net profit. The first term, the valuation of the collected quantity, is the processor's revenue, the only positive cash flow component. Because the processor does not have complete information on the types of collectors within each region, he needs to employ an expectation concept. The second term is the expected transfer payments to the regions that are offered contracts. The third term is the fixed cost of administering the selected regions. The last term is the expected cost of the Stage 2 decisions over all scenarios and is comprised of three cost terms, namely 1) transportation between collection sites to hubs, 2) transportation between hubs and processor, and 3) administration of the selected hubs.

As before, the transportation between hubs and processor can either be rail or truckload, depending on the infrastructure of that particular region. From conversations with trucking companies, rail typically costs more than truck for short distances of below 150 miles.

The first group of constraints imposes the processor's requirement. The quantity constraint (*QC*) requires that the total expected quantity of selected regions exceeds the target quantity. The budget constraint (*BC*) enforces that the total expected lump sum transfer payment of the selected regions is within budget.

The second group of constraints functions as contract construction. The participation constraint (*PC*) requires the least efficient collector of all regions to have nonnegative utility. The incentive constraint set (*IC*) states an acceptable contract should have no worse utility than the contract for other types. As stated in Chapter 3, only adjacent types need to be considered. The monotonicity constraint set (*MC*) enforces that a more efficient collector must have no smaller quantity than the less efficient type.

The third group of constraints considers collectors operational domains. The lower bound quantity constraint (*LQC*) and upper bound quantity constraint (*UQC*) must satisfy all types of collectors.

The fourth group links the first-stage and second-stage decisions. The hub capacity constraint set (*HC*) requires each opened hub to operate within capacity. The accepted efficient constraint (*AEC*) enforces efficient collectors to deliver the required amount of the efficient collector in that particular region. Similarly, the accepted inefficient constraint (*IEC*) enforces similar requirements for inefficient collectors. The logical constraint (*LC*) states that only the hubs in the selected regions can be opened. Moreover, the total number of hubs must be no more than what is available.

The final group enforces correct decision variable types, which are non-negativity and binary.

To study the performance of the SP Lump Sum Model, a numerical study using a few small examples is presented in the following section. After presenting findings from these examples, insights will be presented.

## 6.4 Numerical Study

The parameters in this section do not represent actual data from any industry. This is because the goal was not to implement an industry case study, but to verify that the solution of the SP Lump Sum Model demonstrates reasonable computational behavior. The examples have been solved using the GAMS software with the Baron Solver (www.gams.com).

The inputs to the model probability of acceptance, probability of being efficient, lower and upper bounds of the operating domain, regional fixed cost, and hub fixed cost are all generated from prior knowledge of regional collection activity. Locations of the processor, collectors, and hubs are generated from a uniform distribution. Similarly, the collector and hub capacities are generated with the same distribution so that only the minimum and maximum of these parameters are required. Please refer to Figure 6.5 for the parameter generator flowchart.

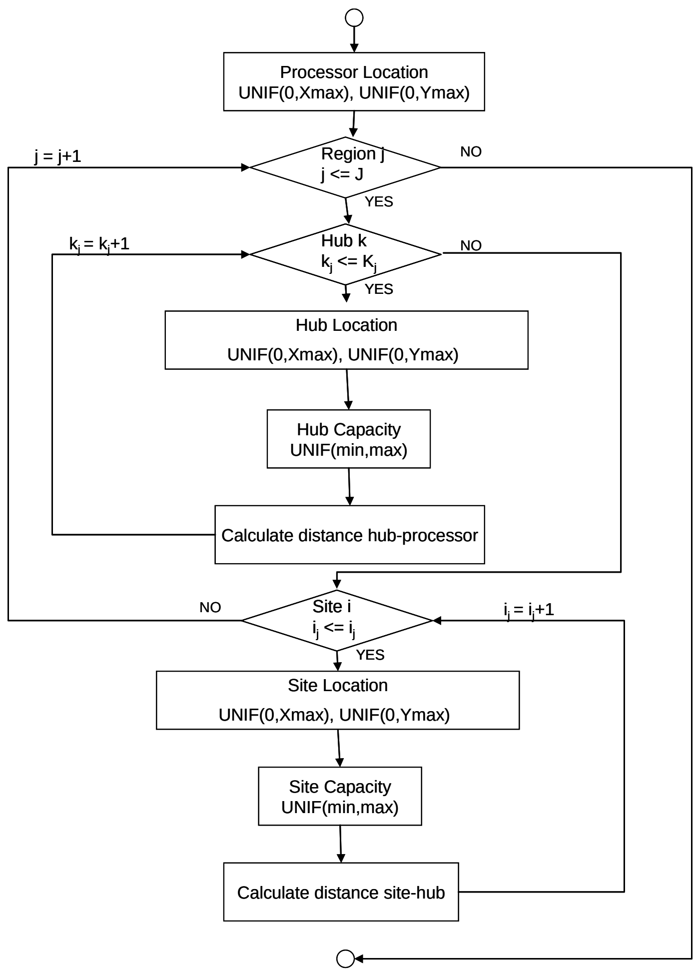



Figure 6.6: Data Generation Flowchart

The numerical study begins with a one region problem and then continues with examples having two and three regions.

### 6.4.1 Example 1

In this first example, there are two regions, one contract (for two collector types), two collectors within each region, and two hubs within each region. Region 1, consisting of Collectors 1 and 2, also contains Hubs 1 and 2. Region 2 contains Collectors 3 and 4 as well as Hubs 3 and 4. Thus, there is a total of 81 scenarios.

The parameters for this example include;

- Efficient marginal cost, $\underline{\theta}$: 0.05 \$/lb, Inefficient marginal cost, $\overline{\theta}$: 0.075 \$/lb
- Efficient collector's range: 200 – 10,000 lb, Inefficient collector's range: 100 – 5,000 lb
- Valuation coefficient: 5, Target quantity: 10,000 lb, Budget: \$2500
- Truck transportation coefficient: 0.01 \$/lb.mile, Rail transportation coefficient (for both Region 1 and 2): 0.006 \$/lb.mile (12 \$/freight.mile, freight = 2000 lb)
- Probability of a collector being efficient for Regions 1 and 2: 0.4 and 0.6, respectively
- Probability of collector accepting a contract for Regions 1 and 2: 0.8 and 0.6, respectively
- Fixed cost for Regions 1 and 2: \$10,000 and \$25,000, respectively
- Fixed cost for Hubs 1, 2, 3, and 4: \$5,000, \$5,000, \$8,000, and \$8,000, respectively
- Distances between Collector 1 to Hubs 1 and 2 are 50 and 100 miles, respectively. Distances between Collector 2 to Hubs 1 and 2 are 120 and 30 miles, respectively. Distances between Collector 3 to Hubs 3 and 4 are 80 and 60 miles, respectively. Distances between Collector 4 to Hubs 3 and 4 are 90 and 20 miles, respectively.
- Distances between Processor to Hubs 1, 2, 3, and 4: 360, 300, 100, 80 miles, respectively
- Capacity for Hubs 1, 2, 3, and 4: 20,000, 20,000, 12,000, and 12,000 lb, respectively

- Probability of scenario: calculated from site independent assumptions. For example (Site1, Site2, Site3, Site4);
  - Scenario 4: (reject, reject, reject, accept-efficient)

    probability (s = 4) = $(1-\pi_1)(1-\pi_1)(1-\pi_2)(\pi_2 p_2)$ = 0.00576
  - Scenario 17: (reject, accept-efficient, accept-inefficient, accept-efficient)

    probability (s = 17) = $(1-\pi_1)(\pi_1 p_1)(\pi_2(1-p_2))(\pi_2 p_2)$ = 0.00553
  - Scenario 41: (accept-efficient, accept-efficient, accept-efficient, accept-efficient) probability (s = 41) = $(\pi_1 p_1)(\pi_1 p_1)(\pi_2 p_2)(\pi_2 p_2)$ = 0.013271
  - Scenario 69: (accept-inefficient, accept-efficient, accept-efficient, accept-inefficient) probability (s = 69) = $(\pi_1(1-p_1))(\pi_1 p_1)(\pi_2 p_2)(\pi_2(1-p_2))$ = 0.013271

The solutions to SP Lump Sum Model of Example 1 include:

- Contract
  - Efficient type ($600, 10,000 lb)
  - Inefficient type ($302, 4,015 lb)
  - Offer contract to both regions ($y_1 = y_2 = 1$)
- Net profit of $108,911
- Stage 2 Policies including hub and allocation decisions (81 scenarios), e.g.,
  - Scenario 4: Open Hub 4. Collect all from collection Site 3 (10,000 lb).
  - Scenario 17: Open Hubs 2, 3, and 4. Site 2 allocates all to Hub 2 (10,000 lb). Site 3 sends 2,014.08 lb to Hub 3 and 20,000 lb to Hub 4. Site 4 transports all to Hub 4 (10,000 lb).

- Scenario 41: Open Hubs 2, 3, and 4. Site 1 transports all to Hub 2 (10,000 lb). Site 2 sends to Hub 2 (10,000 lb). Site 3 allocates 8,000 lb to Hub 3 and 2,000 lb to Hub 4. Site 4 ships to Hub 4 (10,000 lb).
- Scenario 69: Open Hubs 2, 3, and 4. Site 1 ships to Hub 2 (4,014.08 lb). Site 2 sends to Hub 2 (10,000 lb). Site 3 allocates 2,014.08 lb to Hub 3 and 7,985.92 lb to Hub 4. Site 4 transports to Hub 4 (4,014.08 lb)

### 6.4.2 Example 2

In this example, there are two regions, two contracts (four collector types), four collectors within each region, and three hubs within each region. Region 1 consist of Collectors 1, 2, 3, and 4. It also contains Hubs 1, 2, and 3. Region 2 contains Collectors 5, 6, 7, and 8 and Hubs 4, 5, and 6. There is a total of 6561 scenarios.

The parameters include:

- Region 1: Efficient and inefficient marginal costs of $\underline{\theta}$: 0.05 \$/lb and $\overline{\theta}$: 0.075 \$/lb, respectively. Efficient and inefficient collector's range: 400 – 10,000 lb and 100 – 5,000 lb, respectively
- Region 2: Efficient and inefficient marginal costs of $\underline{\theta}$: 0.07 \$/lb and $\overline{\theta}$: 0.09 \$/lb, respectively. Efficient and inefficient collector's range: 300 – 9,000 lb and 150 – 5,500 lb, respectively
- Valuation coefficient: 7.5, Target quantity: 10,000 lb, Budget: \$2500
- Truck transportation coefficient: 0.004 \$/lb.mile
- Rail transportation coefficient: 0.0025 \$/lb.mile (10 \$/freight.mile, 4000 lb/freight)

- Probability of a collector being efficient for Regions 1 and 2: 0.4 and 0.6, respectively
- Probability of collector accepting a contract for Regions 1 and 2: 0.8 and 0.6, respectively
- Fixed cost for Regions 1 and 2: $10,000 and $25,000, respectively
- Fixed cost for Hubs 1, 2, and 3: $5,000 and for 4, 5, and 6: $8,000
- Distances between collector-hub (miles)

| DCH | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 313 | 280 | 507 | 10000 | 10000 | 10000 |
| 2 | 399 | 167 | 610 | 10000 | 10000 | 10000 |
| 3 | 194 | 342 | 324 | 10000 | 10000 | 10000 |
| 4 | 601 | 389 | 792 | 10000 | 10000 | 10000 |
| 5 | 10000 | 10000 | 10000 | 100 | 83 | 627 |
| 6 | 10000 | 10000 | 10000 | 387 | 506 | 202 |
| 7 | 10000 | 10000 | 10000 | 395 | 243 | 946 |
| 8 | 10000 | 10000 | 10000 | 152 | 156 | 738 |

- Distances of hub-processor (miles)

| DHP | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 759 | 504 | 992 | 10000 | 10000 | 10000 |
| 2 | 10000 | 10000 | 10000 | 824 | 987 | 825 |

- Capacity for Hubs 1, 2, and 3: 50,000 lb and for 4, 5, and 6: 80,000 lb

The solutions to SP Lump Sum Model of Example 2 include;

- Contract
  - Region1: Efficient type ( $693, 10,000 lb), Inefficient type ($377, 5,000 lb)
  - Region2: Efficient type ( $608, 8,296 lb), Inefficient type ($14, 150 lb)
  - Offer contract to both regions ($y_1 = y_2 = 1$)
- Net profit of $74,621

In understanding the strategic decisions, the first-stage contract and region decisions are more important that the second-stage hub location and quantity allocation solutions. With dependency to first-stage decisions and high number of scenarios, the second-stage solutions are not reported.

### 6.4.3 Example 3

In this example, there are three regions, two contracts within each region, and six total available hubs. Region 1 consist of Collectors 1, 2, and 3 and Hubs 1, 2, and 3. Region 2 contains Collectors 4, 5, and 6 and Hubs 4 and 5. Region 3 consists of Collectors 7 and 8 with Hub 6. There is a total of 6561 scenarios.

The parameters include;

- Region 1: Efficient and inefficient marginal costs of $\underline{\theta}$: 0.01 \$/lb and $\overline{\theta}$: 0.05 \$/lb, respectively. Efficient and inefficient collector's range: 2,500 – 100,000 lb and 1,000 – 50,000 lb, respectively, 3 collection sites, and 3 hubs
- Region 2: Efficient and inefficient marginal costs of $\underline{\theta}$: 0.04 \$/lb and $\overline{\theta}$: 0.10 \$/lb, respectively. Efficient and inefficient collector's range: 500 – 60,000 lb and 500 – 20,000 lb, respectively, 2 collection sites, and 2 hubs
- Region 3: Efficient and inefficient marginal costs of $\underline{\theta}$: 0.03 \$/lb and $\overline{\theta}$: 0.08 \$/lb, respectively. Efficient and inefficient collector's range: 1,000 – 85,000 lb and 1,000 – 40,000 lb, respectively, 3 collection sites, and 1 hub
- Valuation coefficient: 10, Target quantity: 10,000 lb, Budget: $50,000
- Truck transportation coefficient: 0.004 $/lb.mile

- Rail transportation coefficient: 0.0025 $/lb.mile (10 $/freight.mile, 4000 lb/freight)
- Probability of a collector being efficient for Regions 1, 2, and 3: 0.8, 0.5, and 0.6, respectively
- Probability of collector accepting contract for Regions 1, 2, and 3: 0.8, 0.5, and 0.6, respectively
- Fixed cost for Regions 1, 2, and 3: $100,000, 300,000, and $200,000, respectively
- Fixed cost for Hubs 1, 2, and 3: $25,000 and for 4, 5, and 6: $50,000
- Distances between collector-hub (miles)

| DCH | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 313 | 280 | 507 | 10000 | 10000 | 10000 |
| 2 | 399 | 167 | 610 | 10000 | 10000 | 10000 |
| 3 | 194 | 342 | 324 | 10000 | 10000 | 10000 |
| 4 | 10000 | 10000 | 10000 | 100 | 83 | 10000 |
| 5 | 10000 | 10000 | 10000 | 387 | 506 | 10000 |
| 6 | 10000 | 10000 | 10000 | 10000 | 10000 | 412 |
| 7 | 10000 | 10000 | 10000 | 10000 | 10000 | 77 |
| 8 | 10000 | 10000 | 10000 | 10000 | 10000 | 203 |

- Distances of hub-processor (miles)

| DHP | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 759 | 504 | 992 | 10000 | 10000 | 10000 |
| 2 | 10000 | 10000 | 10000 | 824 | 987 | 10000 |
| 3 | 10000 | 10000 | 10000 | 10000 | 10000 | 928 |

- Capacity for Hubs 1, 2, and 3: 200,000 lb, for 4 and 5: 100,000 lb, and for 6: 250,000 lb.

The solutions (reporting only first-stage) to the SP Lump Sum Model include:

- Contract
  - Region 1: Efficient type ( \$5367, 100000 lb) and Inefficient type (\$4100, 50000 lb)
  - Region 2: Efficient type ( \$4500, 60000 lb) and Inefficient type (\$2000, 20000 lb)
  - Region 3: Efficient type ( \$5200, 83333 lb) and Inefficient type (\$3600, 40000 lb)
  - Offer contract to all regions ($y_1 = y_2 = y_3 = 1$)
- Net profit of \$1,992,473

The results of Examples 1, 2, and 3 raise the question of whether or not quantity contracts can trivially be set to their minimums or maximums. In performing the analysis, one (or a few) parameters at a time are varied to see the resultant solution changes. First considered is the two-region example, followed by an analysis of the three-region example.

### 6.4.4 Analysis of Two-Region Example

There are 6 hubs and 8 total collection sites for a total of 6561 scenarios. The marginal costs are: $\underline{\theta}_1 = 0.05$, $\underline{\theta}_2 = 0.07$, $\overline{\theta}_1 = 0.075$, and $\overline{\theta}_2 = 0.09$. The baseline parameters are directly from Example 2.

The differences in parameters between two consecutive runs are described below (starting with baseline values). The solutions are reported in Table 6.1. The subscript represents the region, *j*. Each constraint is investigated to see whether it is tight, *T*. Decision variables are highlighted if they are at boundary values.

- Run 1: Lower bounds for all quantities are 0 lb, upper bounds of the efficient types are 1,000,000 lb, upper bounds for the inefficient types are 800,000, target quantity is 10,000 lb, and budget is $2500
- Run 2: Target quantity is 10,000 lb and budget is $20,000
- Run 3: Target quantity is 50,000 lb and budget is $3,000
- Run 4: Upper bounds for efficient and inefficient types are 35,000 lb and 20,000 lb, respectively, target quantity is 10,000 lb, and budget is $2500
- Run 5: Lower bounds for efficient and inefficient types are 1,000 lb and 500 lb, respectively, target quantity is 10,000 lb, and budget is $2500
- Run 6: Budget is $3000
- Run 7: Lower bounds for efficient and inefficient types are 1,000 lb and 1,000 lb respectively, upper bounds for efficient and inefficient types are 40,000 lb and 20,000 lb respectively, and budget is $3000
- Run 8: Target quantity is 50,000 lb and budget is $2,500
- Run 9: Target quantity is 40,000 lb
- Run 10: Marginal costs of $\underline{\theta}_1 = 0.05, \underline{\theta}_2 = 0.075, \overline{\theta}_1 = 0.10, \overline{\theta}_2 = 0.125$ ($/lb)
- Run 11: Region 1 has efficient range of 1,000 – 35,000 lb and inefficient range of 500 – 20,000 lb, Region 2 has efficient range of 800 – 30,000 lb and inefficient range of 200 – 15,000 lb

Table 6.1: Solutions to Analysis of Two-Region Example

| RUN | teff1 | qeff1 | tineff1 | qineff1 | teff2 | qeff2 | tineff2 | qineff2 | y1 | y2 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1875 | 37500 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 2 | 2813 | 37500 | 2813 | 37500 | 2813 | 37500 | 0 | 0 | 1 | 1 |
| 3 | 1928 | 37500 | 159 | 2113 | 159 | 2113 | 0 | 0 | 1 | 1 |
| 4 | 1773 | 35000 | 69 | 916 | 69 | 916 | 0 | 0 | 1 | 1 |
| 5 | 1762 | 34638 | 45 | 500 | 80 | 1000 | 45 | 500 | 1 | 1 |
| 6 | 1818 | 35000 | 188 | 2401 | 188 | 2401 | 45 | 500 | 1 | 1 |
| 7 | 2017 | 37500 | 396 | 5085 | 396 | 5085 | 90 | 1000 | 1 | 1 |
| 8 | Infeasible | | | | | | | | | |
| 9 | 1649 | 32188 | 90 | 1000 | 90 | 1000 | 90 | 1000 | 1 | 1 |
| 10 | 1531 | 29125 | 125 | 1000 | 125 | 1000 | 125 | 1000 | 1 | 1 |
| 11 | 1773 | 35000 | 63 | 806 | 63 | 806 | 18 | 200 | 1 | 1 |

| RUN | QC | BC | PC | IC1 | IC2 | IC3 | MC1 | MC2 | MC3 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | T | T | T | T | T | | T | T |
| 2 | | | T | T | T | T | T | T | |
| 3 | | | T | T | T | T | | T | |
| 4 | | T | T | T | T | T | | T | |
| 5 | | T | T | T | T | T | | | T |
| 6 | | T | T | T | T | T | | T | |
| 7 | | T | T | T | T | T | | T | |
| 8 | | | T | T | T | T | | | |
| 9 | | T | T | T | T | T | | T | T |
| 10 | | T | T | T | T | T | | T | T |
| 11 | | T | T | T | T | T | | T | |

It can be observed that the least efficient collector, which is the inefficient collector in Region 2, has a minimum quantity contract value for all runs (if feasible). Moreover, both participation and incentive constraints are tight for all runs (if feasible).

The three-region example was next studied to see whether the solution trend is consistent with that found for the two-region example.

### 6.4.5 Analysis of Three-Region Example

There are 6 hubs and 8 total collection sites with only 10 scenarios considered, by construction, to eliminate unnecessary computation time. The baseline parameters are the same as Example 3. The marginal costs are $\underline{\theta}_1 = 0.01$, $\underline{\theta}_2 = 0.04$, $\underline{\theta}_3 = 0.03$, $\overline{\theta}_1 = 0.05$, $\overline{\theta}_2 = 0.10$, and $\overline{\theta}_3 = 0.08$ \$/lb.

The differences in parameters between two consecutive runs are described below (starting with baseline values). The solutions are reported in Table 6.2 which follows the same notation as Table 6.1.

- Run 1: Region 1 has efficient range of 2,500 – 100,000 lb and inefficient range of 1,000 – 50,000 lb, Region 2 has efficient range of 500 – 60,000 lb and inefficient range of 500 – 20,000 lb, Region 3 has efficient range of 1,000 – 85,000 lb and inefficient range of 1,000 – 40,000 lb, target quantity 100,000 lb, and budget $500,000
- Run 2: Lower bounds are 0 lb for all regions and types, Upper bounds are 100,000 lb for all regions and types.
- Run 3: Target quantity is 400,000 lb and budget is $50,000.
- Run 4: Upper bounds are 500,000 lb for all regions and types.
- Run 5: Upper bounds are 1,000,000 lb for all regions and types.
- Run 6: Coefficient of valuation of 5 $/lb
- Run 7: Coefficient of valuation of 2 $/lb with $y_1 = 1, y_2 = 1, y_3 = 1$.
- Run 8: Coefficient of valuation of 2 $/lb with $y_1 = 1, y_2 = 1, y_3 = 0$.
- Run 9: Coefficient of valuation of 2 $/lb with $y_1 = 1, y_2 = 0, y_3 = 1$.
- Run 10: Coefficient of valuation of 2 $/lb with $y_1 = 0, y_2 = 1, y_3 = 1$

Table 6.2: Solutions to Analysis of Three-Region Example

| RUN | teff1 | qeff1 | tineff1 | qineff1 | teff2 | qeff2 | tineff2 | qineff2 | teff3 | qeff3 | tineff3 | qineff3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 5367 | 100000 | 4100 | 50000 | 4500 | 60000 | 2000 | 20000 | 5200 | 83333 | 3600 | 40000 |
| 2 | 3500 | 100000 | 0 | 0 | 3333 | 83333 | 0 | 0 | 3333 | 83333 | 0 | 0 |
| 3 | 8500 | 100000 | 8333 | 83333 | 8333 | 83333 | 8333 | 83333 | 8333 | 83333 | 8333 | 83333 |
| 4 | 12315 | 500000 | 8148 | 83333 | 8148 | 83333 | 7407 | 74074 | 8148 | 83333 | 8148 | 83333 |
| 5 | 15982 | 1000000 | 6815 | 83333 | 6815 | 83333 | 741 | 7407 | 6815 | 83333 | 6815 | 83333 |
| 6 | 15982 | 1000000 | 6815 | 83333 | 6815 | 83333 | 741 | 7407 | 6815 | 83333 | 6815 | 83333 |
| 7 | 10000 | 1000000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 10000 | 1000000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 10000 | 1000000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | Infeasible | | | | | | | | | | | |

| RUN | y1 | y2 | y3 | QC | BC |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | | |
| 2 | 1 | 1 | 1 | | |
| 3 | 1 | 1 | 1 | | |
| 4 | 1 | 1 | 1 | | T |
| 5 | 1 | 1 | 1 | | T |
| 6 | 1 | 1 | 1 | | T |
| 7 | 1 | 1 | 1 | | |
| 8 | 1 | 1 | 0 | | |
| 9 | 1 | 0 | 1 | | |
| 10 | 0 | 1 | 1 | | |

| RUN | PC | IC1 | IC2 | IC3 | IC4 | IC5 | MC1 | MC2 | MC3 | MC4 | MC5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | T | T | T | T | T | T | | | | | |
| 2 | T | T | T | T | T | T | | T | | T | T |
| 3 | T | T | T | T | T | T | | T | T | T | T |
| 4 | T | T | T | T | T | T | | T | T | T | |
| 5 | T | T | T | T | T | T | | T | T | T | |
| 6 | T | T | T | T | T | T | | T | T | T | |
| 7 | T | T | T | T | T | T | | T | T | T | T |
| 8 | T | T | T | T | T | T | | T | T | T | T |
| 9 | T | T | T | T | T | T | | T | T | T | T |
| 10 | T | T | T | T | T | T | | T | T | T | |

Similar to the previous example, both participation and incentive constraints are tight for all runs (if feasible). However, the most efficient collector, which is the efficient collector in Region 1, has maximum quantity contract value for all runs (if feasible).

### 6.4.6 Insights

The two examples in 6.4.4 and 6.4.5 provide counter-examples to the hypotheses that at the optimal solution (1) the quantity can be set to its upper bound or lower bound is not possible nor are (2) the boundary values are always achieved for the most efficient or least efficient collector.

Due to the complexity of the SP Lump Sum Model, a closed form analysis could not be developed. The aim of the observations from many examples is to reveal patterns of behavior in the solutions.

Since participation and incentive constraints are always tight in the examples (if feasible), tightness of these constraints is therefore assumed so that linking constraints between Stage 1 and Stage 2 are omitted. By setting $\bar{t} = \bar{\theta}\bar{q}$ and $\underline{t} = \bar{t} + \underline{\theta}(\underline{q} - \bar{q})$ for the one region problem, it can be shown that the SP Lump Sum Model is simplified to only target quantity (*QC*), budget (*BC*), and domain constraints. Considering all possible combinations of tightness, the models can be reduced to very small ones. However when $t_4 = \theta_4 q_4$, $t_3 = \theta_3(q_3 - q_4) + \theta_4 q_4$, $t_2 = \theta_2(q_2 - q_3) + \theta_3(q_3 - q_4) + \theta_4 q_4$, and $t_1 = \theta_1(q_1 - q_2) + \theta_2(q_2 - q_3) + \theta_3(q_3 - q_4) + \theta_4 q_4$ for a two region model, the model becomes degenerate. As a result, whether one quantity and lump sum transfer can be set to maximum or minimum values without considering interactions with other quantity and lump sum transfer terms can not be predicted. Moreover, this degeneracy can be extended to problems with more than two regions.

Even with tightness of participation and incentive constraints, it is impossible to declare which contract quantities have minimum or maximum values. Moreover, the above analysis omits Stage 1 and Stage 2 linking constraints. Because a more general model cannot be less complex than a more specific one, extending this finding to a general SP Lump Sum Model cannot trivially predict the contract quantities.

The SP Lump Sum Model offers an additional approach for designing a contract and collection network. With uncertainty in each collector's decision to accept the contract, SP offers complete decisions for all possible scenarios. As mentioned in Chapter 2, the solution to SP is almost never optimal after the fact, but it often is good for all scenarios. However, one major drawback to SP is the high computational requirement. With large numbers of scenarios in typical applications, SP requires the computation for each scenario, which is the opposite of the Simultaneous Lump Sum Model in Chapter 5. One can resolve this high computational effort with Sample Average Approximation, Kleywegt et al. (2001), Shapiro (1996), Shapiro and Homem-de-Mello (1998), and Wei and Realff (2004), for larger problems. This is a possible extension to this research.

In the next chapter, a variable volume type which offers a different transfer payment for different volumes collected, which is truly a very different type of contract to the lump sum structure, is investigated.

# CHAPTER 7

# CONTRACT/NETWORK VARIABLE VOLUME MODELS

Contract and network variable volume models are addressed in this chapter. The chapter begins with the problem description in Section 7.1, followed by the model description in Section 7.2. The variable volume models consist of two stages: the first stage, which determines the Nash Equilibrium contract values, is discussed in Section 7.3, while the second stage, which provides network decisions, is presented in Section 7.4. A case study of Nylon-6 carpet recycling in the southeastern region of the United States is discussed in Section 7.5. Finally, a summary and extensions are presented in Section 7.6.

## 7.1 Problem Description

A typical purchasing quotation in supply management offers varying prices for different order quantities. With larger orders, the supplier can benefit from economies of scale, and hence a lower price per unit. As similar practice is considered in this section for reverse flows, varying price and quantity contracts (variable volume contracts) in RPS have not been explored in literature. The research goal is to gain insights from analyzing simple contract structures.

Unlike the lump sum contract, with a single monetary and quantity point decision, in variable volume cases, a processor proposes a contract structure to the collectors. Each collector decides on the collected quantity from a set of feasible quantities from the processor's proposed structure. A methodology to assist the processor design and evaluate contract options is proposed. These contracts must give incentives to collectors, satisfy the processor's requirements, and ensure the viability of the RPS network.

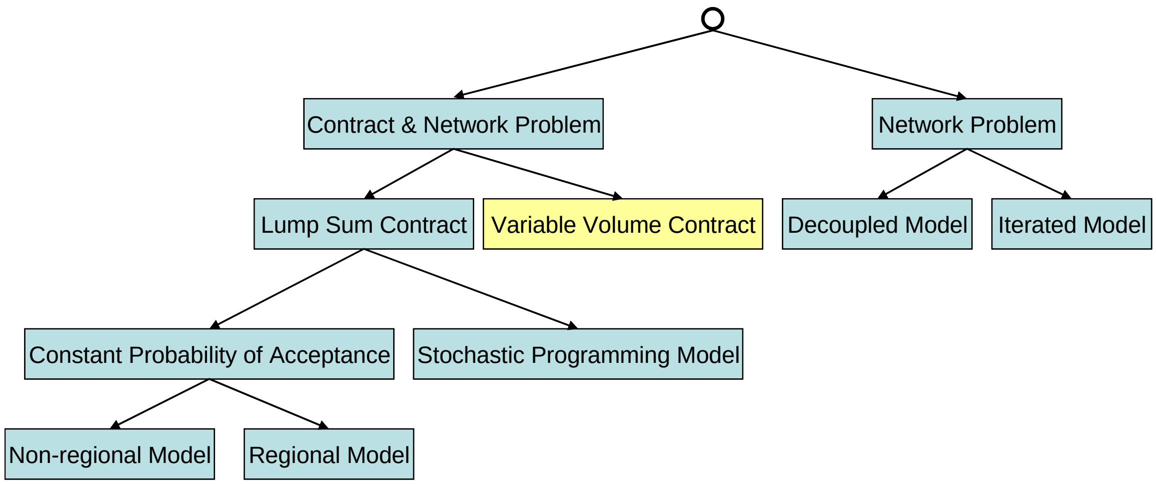



Figure 7.1: Research Tree – Variable Volume Contract

According to the research tree displayed in Figure 7.1, this is the final contract type addressed by my research. The objectives are to 1) analyze each proposed contract structure, 2) understand the needed information to make necessary decisions for each contract structure, and 3) provide insights. Although three simple contract structures are proposed, the basic procedures can be extended to more complicated ones.

## 7.2 Model Description

The Variable Volume Model assists the processor in designing the contract and collection network. This approach, in which the processor gives each collector a different dollar amount depending on the delivered quantity, utilizes a two-stage methodology. The first stage implements the Stackelberg game, Stackelberg (1934), whereas the second stage solves a mathematical model incorporating results from Stage-1 as parameters. Two different models, the Single Collector Type and the Multiple Collector Type, are analyzed. Initially, the Single Collector Type Model with a single global collector type and penalty for deviating from Nash Equilibrium (*NE*) quantities is considered. Subsequently, the Multiple Collector Type Model, which generalizes to many collector types, is studied. The key difference between these two models is the enforcement of the *NE* collection quantity, as discussed in Section 7.4.

The Stage-1 game is categorized as a complete information game because both the processor and collector know each other's payoffs. The known information is consistent with the previous chapters in that the processor has a rough knowledge of the collector's costs (fixed and marginal) and revenue. On the other hand, the collector is aware of the processor cost. With only two players, there are only two moves, namely, one by the processor to specify how much money per pound of used product will be paid, and the second move by the collector to specify the amount of product delivered at that price. As a result, it is also a perfect game. Zermelo (1913) and Kuhn (1953) show that a finite game of perfect information has a pure-strategy *NE*. The backwards induction outcome does not involve non-credible threats. The only sub-game perfect *NE* is the one associated with the backwards induction outcome. It has *NE* for every subgame and history. Therefore, the backwards induction is executed in Stage-1.

Because the *NE* defines the quantity for the collector to provide, the processor must be penalized if he deviates from this quantity. The objective of Stage-2 is to establish a collection network by selecting collectors with specific contracts, a penalty (if there is deviation) from *NE*, fixed administrative, and transportation costs in the processor's net profit maximization.

The original assumptions of one product, one processor and many collectors are still used. Additional assumptions are:

*Assumptions*

- Shipments from collectors directly to the processor (omitting hub cost and allocation)
- Known processor valuation (constant positive slope, i.e. *kv* $/lb)
- All-unit contract
- Single-period setting

With the central focus of analyzing the contract types between processor and collectors, hub cost and allocation are omitted. However, the model can be extended to include this assumption, without loss of generality. The linear processor valuation simplifies the model with each additional collected quantity giving the same marginal revenue. An all-unit contract, in which the collector's total quantity determines the contract type and payment from the processor, is considered. This approach contrasts with the incremental-unit contract which is occasionally implemented in discount production planning models. Finally, a single-period setting with the previous existence of the product is assumed as is commonly the case in remanufacturing literature.

The index, parameters, and decision variables are defined as:

<u>Index</u>

$s$ : Contract structure, $s = 1, 2, 3$

$j$ : Collector type, $j = 1, \ldots, m$

$i_j$ : Collector $i$ of type $j$, $i_j = 1,\ldots,n$ and $n = \sum_j I_j$

For the Single Collector Type Model, $m$ takes the value of one. Additionally, the index $j$ (if applicable) is omitted in all parameters.

Parameters

$Q$ : Processor's target quantity, lb

$B$ : Budget of the processor, \$

$kv$ : Processor's material valuation, \$/lb

$ct$ : Transportation coefficient, \$/lb-mile

$F_{i,j}$ : Fixed cost of collector $i$ of type $j$ to the processor, \$

$d_{i,j}$ : Distance between collector $i$ of type $j$ and the processor, mile

$\theta_j$ : Collector's marginal cost of type $j$, \$/lb

$\beta_j$ : Collector's fixed cost of type j, \$

$a_j$ : Collection cost function coefficient for type $j$, \$/lb$^2$

$b_j$ : Collection cost function coefficient for type $j$, \$/lb

$c_j$ : Collection cost function constant for type $j$, \$

$I_j$ : Total number of collectors of type $j$

Decision Variables

$p_{s,j}$ : Contract revenue coefficient for structure $s$ and type $j$, \$/lb, \$/lb$^2$, or \$/lb$^{0.5}$

$q_{s,j}$ : Contract quantity for structure $s$ and type $j$, lb

$z_{s,j}$ : Structure-Type binary variable

$$z_{s,j} = \begin{cases} 1 & \text{if structure } s \text{ is chosen for collector type } j \\ 0 & \text{otherwise} \end{cases}$$

$y_{s,i,j}$ : Contract offering binary variable

$$y_{s,i,j} = \begin{cases} 1 & \text{if contract structure } s \text{ is offered to collector } i \text{ of type } j \\ 0 & \text{otherwise} \end{cases}$$

*NE* value is denoted with the * operator.

Stage-1, which employs the Stackelberg Economic Model, is to analyze alternative contract structures and solve for both the *NE* quantity and the *NE* contract revenue coefficient. Subsequently in Stage-2, an integer programming (IP) or mixed integer nonlinear programming problem (MINLP) to determine network decisions and actual quantities provided is solved.

The Single Collector Type and Multiple Collector Type Models employ identical Stage-1 calculations. A methodology for solving general *NE* quantity and contract revenue coefficient is presented. Specifically, there are three alternative structures under consideration: the contract with linear, square, and square root contract revenue structures. There are additional constraints that are standard for the Stackelberg Economic Model. The Stage-1 Model is presented in the following section.

## 7.3 Variable Volume Contract Model: Stage-1

Stage-1 aims to find the *NE* for the collector's quantity and the processor's price. The Stackelberg Model can be summarized in Figure 7.2. It is a pair-wise game between the processor and the collector who is a representative collector for the interested group, i.e., all collectors in the group have the same cost function and operate in the same manner. First, the processor offers the contract type to the collector. Then the collector responds with the collected quantity.

Backward induction is used to find the *NE*. As mentioned, the backward induction eliminates all the non-credible threats. Let $g(q, p)$ be the contract function, * be *NE* value, $n$ be total number of collectors, $b$ be capacity buffer, and $Q$ be the processor's target quantity. The backward induction steps are summarized in Procedure 7.1.

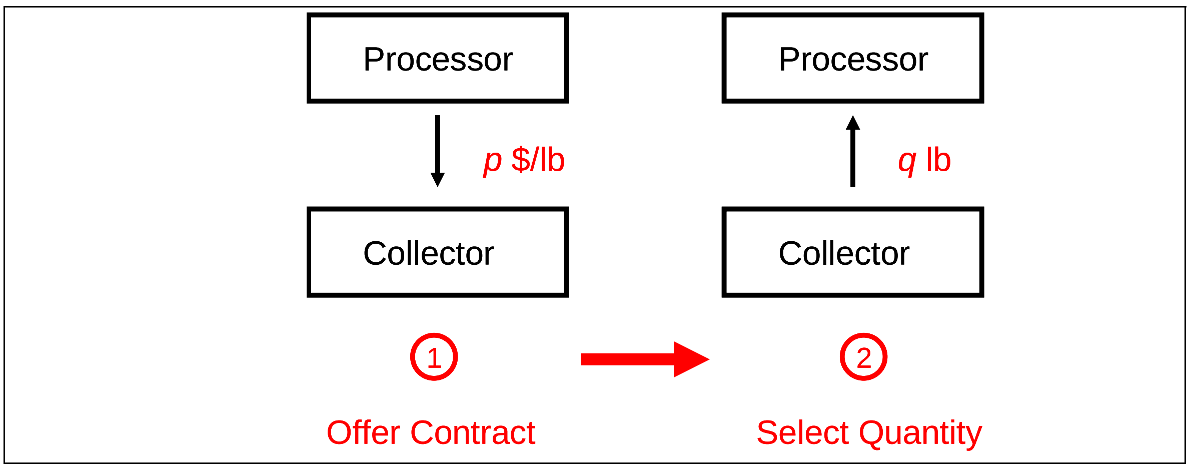



Figure 7.2: Step 1 of the Variable Volume Contract Model

**<u>Procedure 7.1: Stage-1 Backward Induction</u>**

1. The collector determines the best response, in terms of quantity $q$, to processor's action by solving:

$$\textbf{Max}_{\mathbf{q}} \quad \pi_c : \text{Collector's Net Profit} = \text{Revenue} - \text{Cost} = g(q, p) \text{ - Cost}$$

$$\textbf{s.t.} \quad \pi_c \geq 0$$

$$q_{\min} \leq q \leq q_{\max}$$

2. Substituting the collector's best response into the processor's problem, the processor determines his best action, in terms of revenue coefficient $p$, by solving:

$$\textbf{Max}_{\mathbf{p}} \quad \pi_p : \text{Processor's Net Profit} = kv.q^* - g(q^*, p)$$

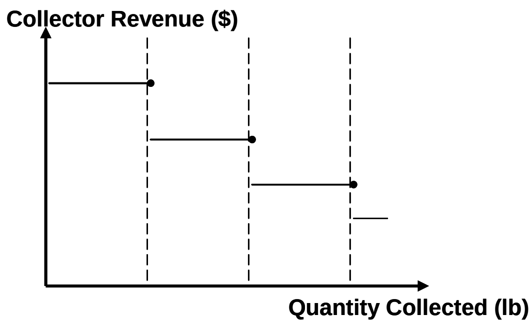
$$\textbf{s.t.} \quad nq^* \geq (1+b)Q$$

$$\pi_p \geq 0$$

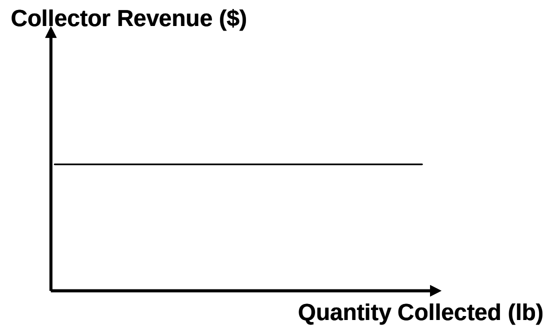
$$p \geq 0$$

The Stage-1 Model finds *NE* contract values, $p^*$ and $q^*$, for different contract structures. In some cases, there are multiple *NE* solutions. The processor has the first-mover advantage to select the *NE* solution that produces the highest processor's net profit. Possible contract structures are analyzed with the use of the collector's revenue curve. The revenue curve represents what the processor pays to the collector, which is equivalent to how much the collector receives. The vertical axis has units of dollars and the horizontal axis has units of pounds. For example, the lump sum contract has two points representing contracts for efficient and inefficient collectors.

A key requirement for the revenue curve is revenue increases as collected quantity increases. Figure 7.3 gives examples of infeasible revenue curves. Figure 7.3a) is infeasible because it has negative slope. As the collector obtains higher quantity, he earns less revenue. Similarly, the collector does not gain any additional revenue by collecting more material as shown in Figure 7.3b).



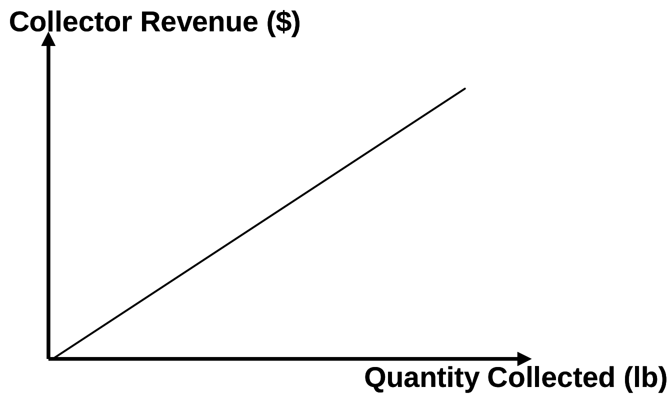

*a)*




*b)*

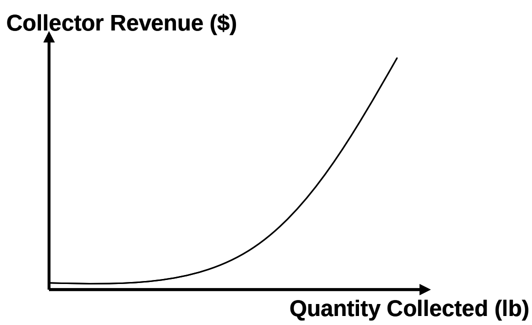
Figure 7.3: Examples of Infeasible Revenue Curves

In considering contract structures, three structures are studied. Figure 7.4 displays these contract structures with their revenue terms. Structure 1 employs a constant specified $/lb revenue. Structure 2 has an increasing $/lb revenue as quantity increases. Lastly, Structure 3 has a decreasing $/lb revenue as quantity increases. These three structures are selected because they are intuitive and represent three broad categories of contracts. These broad categories are linear (both convex and concave), convex, and concave structures. By exploring these structures, the processor has the ability to select different types of contract with knowledge of valuation and cost coefficients. The linear contract, Figure 7.4a), is the default contract structure because it is intuitive and simple to the processor. Hence, if there is no special reason for considering a concave or convex revenue contract, it is recommended to offer the linear contract structure.



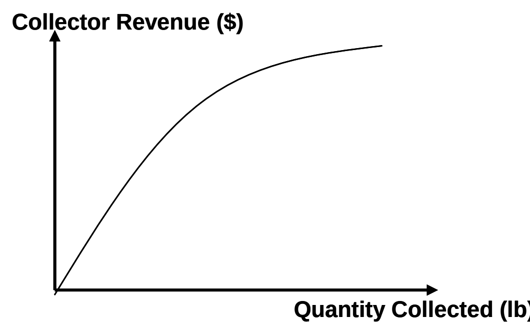

Revenue = $pq$, where $p$: \$/lb

*a)* Linear Contract Structure




Revenue = $pq^2$, where $p$: \$/lb$^2$

*b)* Convex Contract Structure




Revenue = $p\sqrt{q}$, where $p$: \$/lb$^{1/2}$

*c)* Concave Contract Structure

Figure 7.4: Studied Collector's Revenue Structures

The other term in the collector's net profit maximization model is the collection cost. A representative collector from a particular group has a total cost representation of $TC = \theta q + \beta$. The original assumption that the processor knows the collector's marginal cost, $\theta$, still holds. Additionally, it is assumed that the collector's fixed cost, $\beta$, is also roughly known. This linear total cost requires relatively little information. The processor should already have these approximate values if he is thinking about establishing a collection network. If not, he should be able to rank a particular collector to already known collectors to construct this fixed cost. If there is no way to approximate this information, this collector should not be considered due to high uncertainty and risk.

For the linear revenue contract structure, the collector's revenue and processor's cost functions are $pq$ with the collector's cost of $\theta q+\beta$. Performing Procedure 7.1 for the collector and processor problems, the *NE* solutions are: 1) $q^*=q_{\max}$ and 2) $p^*=\frac{\beta}{q_{\max}}+\theta$ with the condition that $kv\geq p^*\geq\theta$.

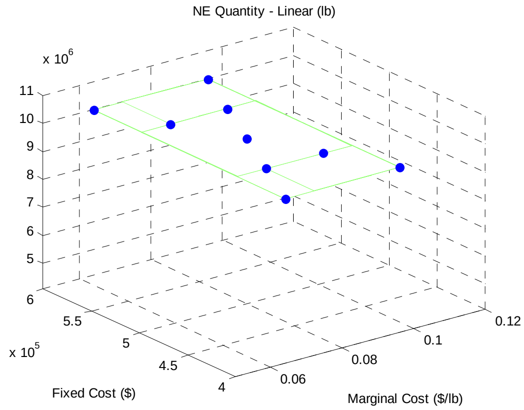
For a square revenue contract structure, the collector's revenue and processor's cost have the function of $pq^2$ and the collector's cost term equals $\theta q+\beta$. Performing Procedure 7.1, all possible cases produce no *NE* solutions. The processor wants to offer a revenue coefficient of zero, which forces the collector to defect. The intuitive explanation is that as the revenue term grows rapidly, the collector wants to collect a very high quantity. However, at this high quantity level, the processor incurs an extremely high contract cost with only linear valuation. Therefore, the processor and collector have a conflict of interest.

Finally for the square root revenue contract structure, the collector's revenue and processor's cost have the function of $p\sqrt{q}$ and the collector's cost term equals $\theta q+\beta$. Performing Procedure 7.1 to the collector and processor problems, the *NE* solutions are: 1) $q^*=\frac{\beta}{\theta}$ and 2) $p^*=2\sqrt{\theta\beta}$.

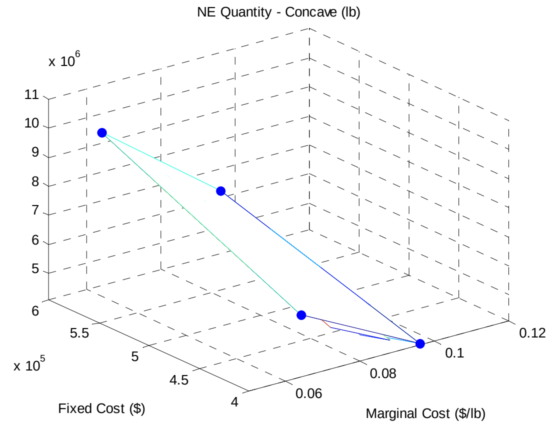
After obtaining the *NE* solution by setting the first order condition to zero, one needs to check whether the solution produces a maximum. The violations of constraints can provide helpful suggestions to the processor. When $nq^* < (1+b)Q$, the processor needs to expand the collector list or adjust $Q$. Both $q_{min}$ and $q_{max}$ are related to technology advancement. When the *NE* quantity falls below $q_{min}$, the processor needs to encourage collectors to reduce the set up cost or explore new technology. On the other hand, when the *NE* quantity exceeds $q_{max}$, the processor should encourage collectors to expand or explore new technology.

To understand the impact of changes in $\theta$ and $\beta$, sensitivity analysis is performed. As a subject in an analysis, a collector with marginal cost of \$0.08/lb, fixed cost of \$500,000, $q_{min}$ of 1M lb, and $q_{max}$ of 10M lb is chosen. What if $\theta$ and $\beta$ are inaccurate by $\pm 10\%$ and $\pm 20\%$? How will the $p^*$, $q^*$, and contract revenue vary for linear and concave contracts? Figure 7.5 displays sensitivity plots. I only consider $\theta$ and $\beta$ values within their feasible regions.

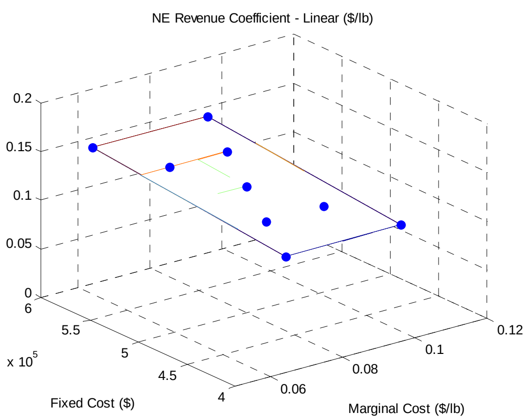
For the linear contract, $q^*$ is independent to changes in $\theta$ and $\beta$. As $\theta$ varies, $p^*$ moves in the same direction by the same magnitude (slope of one). Although $\beta$ influences $p^*$ in the same manner, it is less sensitive. With high $q_{max}$ value, $\partial p^* / \partial \beta$ has a small impact. Finally, the contract revenue behaves in a relatively linear manner with changes in $\theta$ and $\beta$, as depicted in Figure 7.5e). As long as $kv \geq \theta$, having an accurate marginal cost is much more crucial than a fixed cost. Because the $q^*$ is driven to $q_{max}$, the collector is not concerned about $p^*$; therefore the processor only needs to have $p^*$ less than $kv$. For the linear contract, getting $\theta$ close to the real marginal cost for each collector type is important.



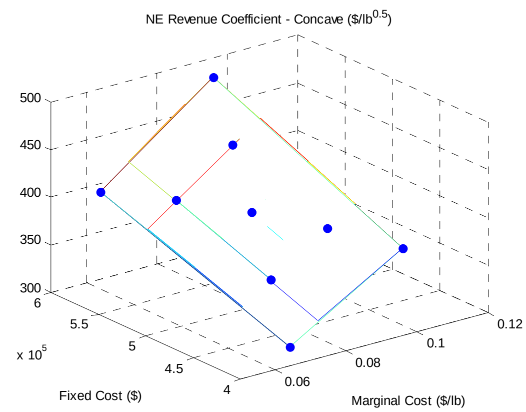

a) $q*$ - Linear



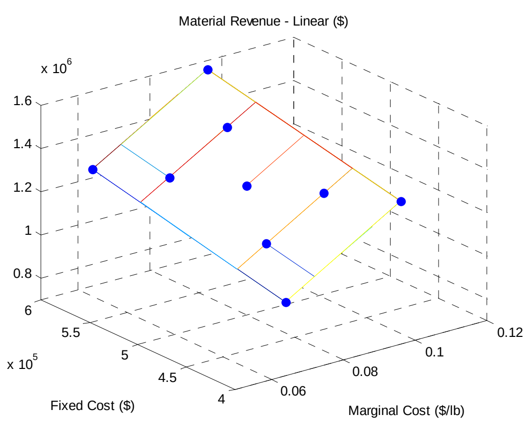

b) $q*$ - Concave



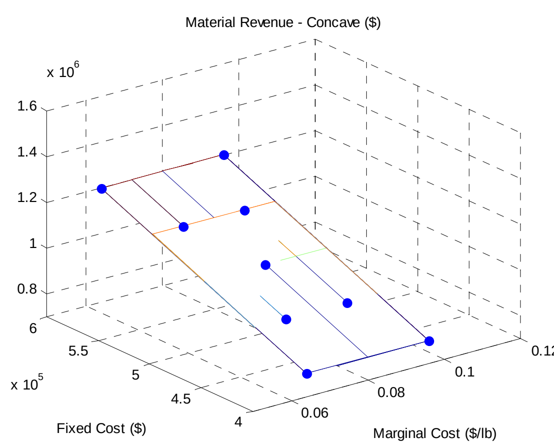

c) $p*$ - Linear




d) $p*$ - Concave




e) Material Revenue – Linear




f) Material Revenue – Concave

Figure 7.5: Sensitivity Plots for $\theta$ and $\beta$

For the concave (square root) contract, both $q^*$ and $p^*$ depend on $\theta$ and $\beta$. The *NE* quantity has inverse relationship with marginal cost, but has positive linear relationship with fixed cost. Although the changes in $\theta$ and $\beta$ affect $q^*$ roughly equally when the percentage is relatively low, $\beta$ has significantly stronger impact at high percentage changes. With a square root function, little changes in $\theta$ or $\beta$ have smaller impact to the $p^*$ than of the linear contract. Moreover, changes in $\theta$ or $\beta$ have the exact same impact to the *NE* revenue coefficient. Considering both changes in $\theta$ and $\beta$, the sensitivity to the contract revenue can be seen in Figure 7.5f). One can see that changes in marginal cost do not affect the contract revenue. Unlike the linear contract where $\theta$ influences more changes, having $\beta$ close to their true values is important in the concave contract.

Although the linear collector total cost is recommended and implemented in Section 7.5, a u-shaped marginal cost is also considered in order to show that different collector total costs greatly influence the *NE* solutions. The u-shaped marginal cost can be algebraically represented by a parabolic curve of $C(q) = aq^2 + bq + c$, when $q \geq 0$. Figure 7.6 shows a typical collector's marginal cost curve. The requirements for the cost function are 1) $c$ is a positive value, i.e. the fixed set up cost is non-negative, 2) $a$ is positive, i.e. a convex cost curve is required, and 3) b is non-positive to require that the minimum cost quantity is positive or to the right of the y-axis, i.e. $-\frac{b}{a} \geq 0$.

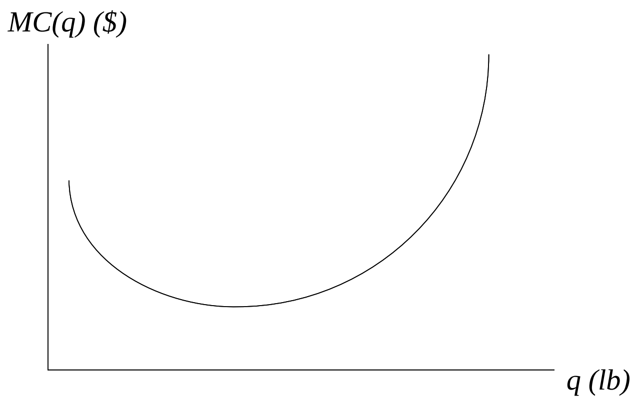


Figure 7.6: Collector's Marginal Cost Function for the Variable Volume Contract

From this u-shaped marginal cost curve, the total cost can be derived by integration. The total cost can be written as $TC(q) = aq^3 + bq^2 + cq$. There are four solution pairs for the linear and square contract structures. However, there are no closed form solutions for the square root contract, hence it is necessary to solve for the *NE* solutions numerically. Because solutions to the polynomial cost function are very lengthy and are not used for later analyses, they are presented in Appendix B. By having a more complicated total cost function, the processor could find more accurate solutions but with the tradeoff of additional complication. With a higher order of total cost function, there are likely to be multiple solutions, so that a tie-breaking rule must be used. However with approximated cost parameters, a more complicated total cost function may not give any advantages. Therefore, the linear total cost function is preferred and is used in the case study in Section 7.5.

In Stage-1, the *NE*s quantity and revenue coefficient for each of the three structures are obtained. When the processor offers a contract specifying the revenue coefficient to the collector, the collector makes a quantity selection. The dynamic game with complete and perfect information is modeled with the Stackelberg game. As a result of the backward induction, the *NE* solutions of $q^*$ and $p^*$ are selected. Neither the processor nor the collector will want to deviate from these values. Having all the needed parameters, Stage-2 models are next discussed in the following section.

## 7.4 Variable Volume Strategic Network Model: Stage-2

The objective of Stage-2 is to construct the collection network. By incorporating decisions from Stage-1, mathematical programming models are developed to select collectors to join the processor's collection network. Figure 7.7 displays the Stage-2 problem. In addition to the contract payment, the processor needs to pay for fixed administrative and transportation costs for each chosen collector. The processor has to decide which collectors to offer the contract in maximizing his net profit. Again, the key assumption is that there is a representative collector. All collectors in the same group have the same cost function. From the Stage-2 model, the processor can maximize his net profit and analyze his cost components.

Two Stage-2 network models are studied. The initial model is the Single Collector Type Model which considers a single global collection type. Additionally, the concept of penalizing the processor for not implementing *NE* quantities is utilized. Following the study of the Single Collector Type Model, the Multiple Collector Type Model is analyzed and becomes the final and proposed Stage-2 model because it extends to many collector types. The concept of fixing the *NE* quantities is applied here. Although the Single Collector Type Model is superceded in generality by the Multiple Collector Type Model, it is presented in the following section for completeness.

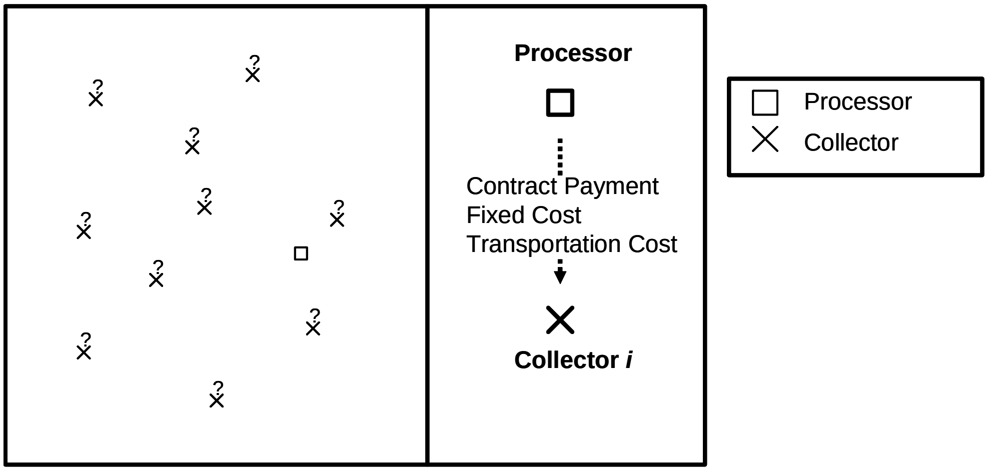


Figure 7.7: Stage-2 Problem Description

### 7.4.1 Single Collector Type Model

In this model, the *NE* revenue coefficient, *p**, is fixed going into the Stage-2 model. Since the processor is the leader (more powerful), it is assumed that he can fix this *p** value. However, he is allowed to deviate from the *NE* quantity in this model. With additional network constraints, the processor may obtain higher net profit by moving away from *q**.

In running and analyzing the model, I found that there are numerous examples where the processor chooses to deviate from *q**, which directly reduces the collector's net profit. To adjust for this deviation, a lump sum penalty payment from the *NE* quantity for each collector is attached. This correction is very important to encourage collectors to join the network. Two proposed lump sum penalty forms are:

- linear in deviation, $lp\,|\,q_i - q_i^*\,|$, where *lp* is constant \$/lb
- quadratic in deviation, $lp(q_i - q_i^*)^2$, where *lp* is constant \$/lb$^2$

The difference between these two forms is the marginal penalty. The quadratic penalty function penalizes marginally less for a small deviation and marginally higher when the processor deviates greatly from the *NE* quantity. On the other hand, the linear penalty function penalizes equally per unit deviated. For the Single Collector Type Model, the quadratic lump sum penalty is chosen to deter the processor from significantly deviating from $q^*$. Examples of quadratic penalty function include Baker and Scudder (1990) and Schaller (2004). For this Single Collector Type Model, individual Stage-1 and Stage-2 calculations are performed for each contract structures. In the case of contract structures, there are three different Step-2 calculations. Figure 7.8 displays the Single Collector Type Model.

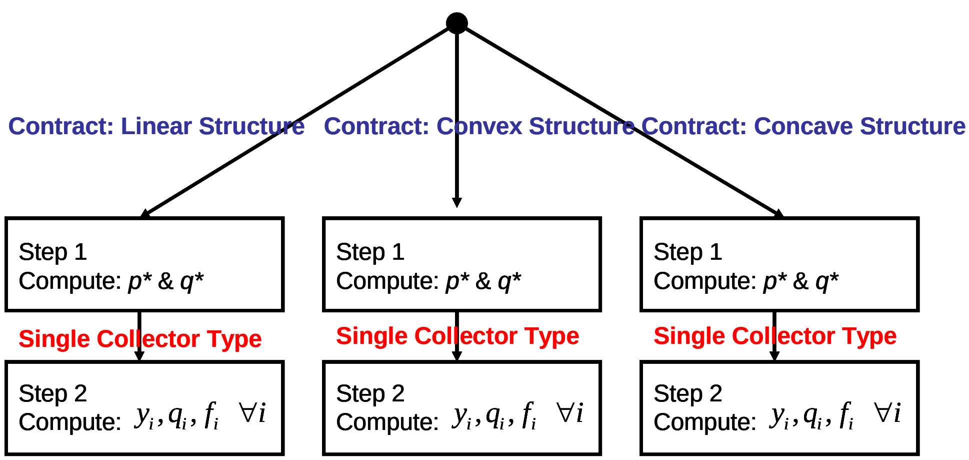



Figure 7.8: Single Collector Type Model (Stage-2)

Letting *i* be the collector index, additional notations include:

| | | |
|---|---|---|
| $lp$ | : | Lump sum penalty coefficient, \$/lb$^2$ or \$/lb |
| $\Delta$ | : | Profit to be shared among collectors, \$ |
| $f_i$ | : | Fraction of the profit to be shared to collector $i$ |

$SE$ : Shared excess profit fraction to collectors, $0 \le SE \le 1$

$m$ : Number of collectors at NE quantity needed to obtain total quantity $Q$, $Q/q^*$

$m_D$ : $\lfloor Q/q^* \rfloor$

$m_U$ : $\lceil Q/q^* \rceil = m_D + 1$

$\pi_Q$ : Net Profit when processor obtains total quantity $Q$, \$

$\pi_D$ : Net Profit when processor obtains quantity of $m_D q^*$, \$

$\pi_U$ : Net Profit when processor obtains quantity of $m_U q^*$, \$

The profit to be shared among collectors, $\Delta$, is an excess profit from the processor deviating from the *NE* quantity to obtain the target quantity, $Q$. Because there is only a single collector deviating from $q^*$, the $\pi_D$ and $\pi_U$ are crucial in determining $\Delta$. Once the excess profit is determined, its fractions are to be shared among collectors. The processor can decide on the maximum total fraction to be shared by setting his *SE* parameter.

To obtain $\pi_D$ and $\pi_U$, one needs to solve the unconstrained objective function of maximizing $kv\sum_i q_i - p^*\sum_i g(q_i) - ct\sum_i d_i q_i - \sum_i F_i y_i$ for the *NE* quantity for each collector, where $g(q_i) = q_i$, $g(q_i) = (q_i)^2$, and $g(q_i) = \sqrt{q_i}$ for structure $i$ = 1, 2, and 3 respectively. By choosing $m_D$, collectors with the best combinations of $F_i$ and $d_i$, $\pi_D$ can be obtained. Similarly, by choosing $m_U$ collectors with the best combinations of $F_i$ and $d_i$, $\pi_U$ can be found. Finally, the profit to be shared among collectors, due to deviation from *NE*s, can be from $\Delta = \min_{s.t.\ge 0}\{\pi_Q - \pi_D, \pi_Q - \pi_U\}$. To achieve the target quantity, $Q$, one collector must collect either more than $q^*$ or less than $q^*$. Since the processor gets penalized for this deviation from $q^*$, the processor must choose the minimum between the positive value(s) of $\pi_Q - \pi_D$ and $\pi_Q - \pi_U$ to be the excess profit sharing among collectors. To calculate $\pi_Q$, one solves:

$$\pi_Q : kv * Q - \min\ p_i^* \sum_i g(q_i) + ct \sum_i d_i q_i + \sum_i F_i y_i$$

$$\text{s.t.} \quad \sum_i q_i = Q$$

$$q_i \leq Q y_i \qquad \forall i$$

With indices, parameters, and decision variable defined, the Single Collector Type Model can be written as follows:

**Single Collector Type Model**

Max $\quad kv \sum_i q_i - p_i^* \sum_i f(q_i) - ct \sum_i d_i q_i - \sum_i f_i \Delta - \sum_i F_i y_i \qquad$ *(OBJ)*

s.t. $\quad \sum_i q_i = Q \qquad$ *(QC)*

$p^* \sum_i g(q_i) + \sum_i f_i \Delta \leq B \qquad$ *(BC)*

$[p^* f(q_i) + f_i \Delta - c(q_i)] y_i \geq 0 \qquad \forall i \qquad$ *(PC)*

$lp(q_i - q^*)^2 \leq f_i \Delta \qquad \forall i \qquad$ *(LPC)*

$\sum_i f_i \leq SE \qquad$ *(TLPC)*

$q_{\min,i} y_i \leq q_i \qquad \forall i \qquad$ *(LBC)*

$q_i \leq q_{\max,i} y_i \qquad \forall i \qquad$ *(UBC)*

$q_i \geq 0, f_i \geq 0\ , y_i \in \{0,1\} \quad \forall i$

The *OBJ* includes the additional cost of a penalty cost. As the fraction of excess profit for each collector is found, the summation is a cost term for the processor. The *QC* and *BC* are the original target quantity and budget constraints, whereas the *PC* constraint enforces participation. Each collector needs to have a nonnegative net profit. The *LPC* constraint is the lump sum penalty constraint for each collector with the quadratic function. *TLPC* enforces the total penalty fraction to be less than *SE*, which is the processor's parameter of value between zero and one. Finally, the *LBC* and *UBC* define the minimum and maximum quantities.

This model assists the processor in deciding 1) who to offer the contract to, 2) the amount of collected quantity for each collector, and 3) the fraction of profit to be shared by each collector.

The key drawback of this model is that the *NE* values are not enforced. Although a penalty is implemented to give an incentive to collectors, the issue of fairness is still in doubt. Moreover, this is a MINLP model with a non-intuitive lump sum penalty coefficient. Although the Single Collector Type Model is not the proposed model, it is an initial and alternative approach. The Multiple Collector Type Model extends to many collector types with enforcement of *NE* quantities and is presented in the following section.

### 7.4.2 Multiple Collector Type Model

In this model, both the *NE* quantity and *NE* revenue coefficient are fixed into the Stage-2 model. The major extension from the previous model is the multiple collector types. Because population, local recycling laws, and urban density are key factors in strategic collection design, they are embedded in the collector type, *j*. This model can be summarized in Figure 7.9.

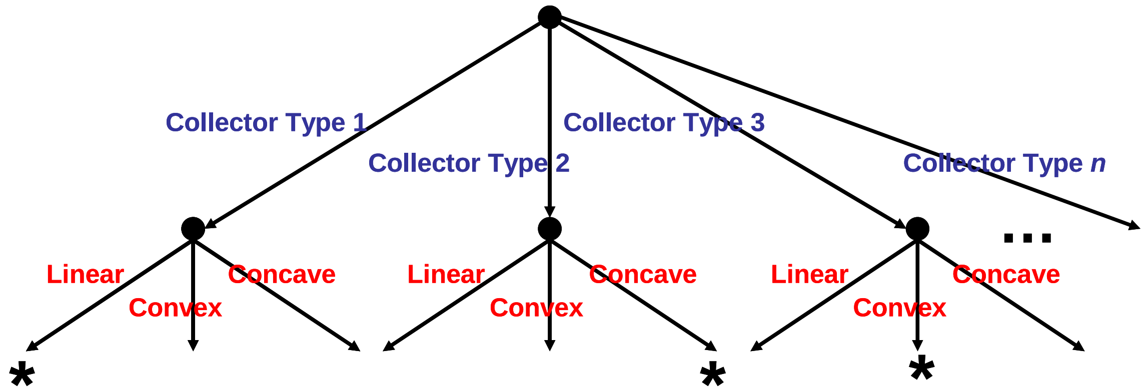


Figure 7.9: Multiple Collector Type Model (Stage-2)

Two major decisions are determined which are 1) which contract structure to offer each collector type and 2) given the determined structure, which collector to offer the contract. As shown in Figure 7.9, the star represents the selected contract structure for each collector type. The model is an integer programming (IP) mathematical model, which is easily solved by any commercial optimization software. With $p^*_{s,j}$ and $q^*_{s,j}$ from the Stage-1 model as input parameters, the model can be written as:

**Multiple Collector Type Model**

Max
$$kv\sum_j\sum_{i_j}\sum_s q^*_{s,j}y_{s,i,j}-\sum_s\sum_j p^*_{s,j}\sum_{i_j}g_s(q^*_{s,j}y_{s,i,j})-ct\sum_j\sum_{i_j}\sum_s d_{i,j}y_{s,i,j} \qquad (OBJ)$$
$$-lp.(\sum_j\sum_{i_j}\sum_s q^*_{s,j}y_{s,i,j}-Q)-\sum_j\sum_{i_j}\sum_s F_{i,j}y_{s,i,j}$$

s.t.
$$\sum_j\sum_{i_j}\sum_s q^*_{s,j}y_{s,i,j}\geq Q \qquad (QC)$$
$$\sum_s\sum_j p^*_{s,j}\sum_{i_j}g_s(q^*_{s,j}y_{s,i,j})+lp.(\sum_j\sum_{i_j}\sum_s q^*_{s,j}y_{s,i,j}-Q)\leq B \qquad (BC)$$
$$\sum_s z_{s,j}=1 \qquad \forall j \qquad (SC)$$
$$\sum_{i_j} y_{s,i,j}\leq I_j z_{s,j} \qquad \forall s,j \qquad (LC)$$
$$z_{s,j}\in\{0,1\},\quad y_{s,i,j}\in\{0,1\}$$

The OBJ function includes the processor's valuation, contract payments, transportation, penalty for exceeding target quantity, and fixed administrative costs to the collectors. The QC and BC are the original target quantity and budget constraints. The SC is the structural constraint set that forces each collector type to select the best contract structure. Finally, LC is the logical constraint. If the contract structure is not the best for the collector type, then LC prevents that type of contract from being offered. If it is the best contract structure, the maximum number of collectors selected is $I_j$. The Multiple Collector Type Model is utilized in the case study which is discussed in the following section.

## 7.5 Carpet Case Study

This case study aims to test the Variable Volume Model in an industrial scale environment with representative cost structure. Nylon-6 carpet recycling in the southeast of the United States is considered. The states of interest include Alabama, Florida, Georgia, North Carolina, South Carolina, and Tennessee. As mentioned in Assavapokee et al. (2007), the key issue in carpet recycling is the recovery and classification of the carpet from consumers. This is normally facilitated by installers who return carpets to the retail store after they have been removed from buildings directly to a recycling facility, avoiding an expensive curb-side recycling system.

The known information of the collectors remains identical to the previous chapters. The processor only has knowledge of the marginal cost, $\theta$ \$/lb, and the fixed cost, $\beta$ \$, of each collector. The total cost function of collector $i$, $TC_i$, can be mathematically written as $TC_i = \theta_i q + \beta_i$ where $q$ is the collected quantity. Two grouping designs are implemented to understand the effect of grouping. Only the annual carpet volume is considered in the first grouping design. On the other hand, the state in consideration and its annual volume categorize the second grouping design.

All data, which are representative of an industry instance, are from Assavapokee et al. (2007) and Ashman (2007). The latter has significant carpet recycling experience in the northeast region of the United States. The data are presented in Section 7.5.1. The two grouping methods are explained in Section 7.5.2. The application of the two-stage Variable Volume Model to the case study follows in Section 7.5.3. Finally, the results are presented in 7.5.4 with conclusions.

### 7.5.1 Data

The large recycling processing site being considered for opening is in Atlanta, Georgia. The processor wants to establish a collection network for the southeast region of the United States. The first set of data contains the collectors' volumes and locations. Although referred to as the collection site, it is an aggregation of point sources in a given region. Hence a collection site in Tampa, Florida consists of many smaller point sources within that region. Due to the confidentiality of the data, the annual volume for each collection site is not displayed. Instead, only the minimum and maximum annual volumes are presented without explanation. The 100 sites with the highest annual volumes are selected for the case study as seen in Table 7.1. These sites are ranked by volume in descending order with the distances from the processing site displayed.

The processing facility needs to obtain approximately 80 million lb of Nylon-6 carpet annually with a material budget of \$5.6M. This budget results from the processor's target price of \$0.07/lb. Each pound of (mechanically) processed Nylon-6 carpet (resin pallet form) is valued at \$0.28/lb by the processor. With an assumption of \$0.10/lb processing cost, the collected Nylon-6 carpet has valuation of \$0.18/lb. The coefficient of penalty for exceeding the target quantity is \$0.20/lb. The transportation mode considered is trucking with a variable cost of \$3/load-mile. The truck capacity baled is 44,000 lb/load. Hence, the coefficient of transportation cost has a value of \$0.0000682/lb-mile (\$0.1364/ton-mile).

Table 7.1: Collection Sites Information

| Site | CityName | Distance (miles) | Qmin | Qmax | Site | CityName | Distance (miles) | Qmin | Qmax |
|---|---|---|---|---|---|---|---|---|---|
| 1 | TAMPA, FL | 416 | 10600490 | 35334967 | 26 | BIRMINGHAM, AL | 138 | 1793548 | 5978494 |
| 2 | ORLANDO, FL | 405 | 9645417 | 32151389 | 27 | MARIETTA, GA | 15 | 1583646 | 5278820 |
| 3 | MIAMI, FL | 605 | 7424016 | 24746720 | 28 | COLUMBIA, SC | 204 | 1548913 | 5163044 |
| 4 | POMPANO BEACH, FL | 579 | 6780090 | 22600301 | 29 | PENSACOLA, FL | 280 | 1417765 | 4725884 |
| 5 | JACKSONVILLE, FL | 287 | 6317454 | 21058181 | 30 | DEERFIELD BEACH, FL | 574 | 1365326 | 4551088 |
| 6 | CHARLOTTE, NC | 227 | 5668796 | 18895987 | 31 | EATONTON, GA | 67 | 1351948 | 4506492 |
| 7 | FORT LAUDERDALE, FL | 584 | 4585926 | 15286419 | 32 | WEST PALM BEACH, FL | 547 | 1332169 | 4440563 |
| 8 | ATLANTA, GA | 10 | 4471494 | 14904981 | 33 | NORTH CHARLESTON, SC | 260 | 1295781 | 4319271 |
| 9 | KNOXVILLE, TN | 155 | 3988330 | 13294434 | 34 | NAPLES, FL | 548 | 1231403 | 4104676 |
| 10 | MEMPHIS, TN | 332 | 3988197 | 13293990 | 35 | MOBILE, AL | 302 | 1168327 | 3894425 |
| 11 | NORCROSS, GA | 17 | 3278221 | 10927403 | 36 | CHATSWORTH, GA | 73 | 1164119 | 3880398 |
| 12 | GREENSBORO, NC | 306 | 3146657 | 10488856 | 37 | HIALEAH, FL | 598 | 1158130 | 3860434 |
| 13 | FORT MYERS, FL | 515 | 2872247 | 9574155 | 38 | MORRISVILLE, NC | 348 | 1152852 | 3842839 |
| 14 | SARASOTA, FL | 457 | 2732003 | 9106678 | 39 | HUNTSVILLE, AL | 142 | 1146800 | 3822665 |
| 15 | RALEIGH, NC | 357 | 2678636 | 8928788 | 40 | SAINT PETERSBURG, FL | 427 | 1137970 | 3793235 |
| 16 | DULUTH, GA | 23 | 2440090 | 8133633 | 41 | DORAVILLE, GA | 13 | 1110948 | 3703161 |
| 17 | CLEARWATER, FL | 410 | 2198842 | 7329475 | 42 | WILMINGTON, NC | 375 | 1110919 | 3703062 |
| 18 | LONGWOOD, FL | 393 | 2158210 | 7194032 | 43 | GALLATIN, TN | 214 | 1097768 | 3659227 |
| 19 | NASHVILLE, TN | 213 | 2155604 | 7185346 | 44 | FAYETTEVILLE, NC | 328 | 1092558 | 3641859 |
| 20 | OCALA, FL | 343 | 2072900 | 6909667 | 45 | PORT SAINT LUCIE, FL | 508 | 1051914 | 3506381 |
| 21 | DURHAM, NC | 348 | 2001805 | 6672685 | 46 | TALLAHASSEE, FL | 228 | 1048123 | 3493743 |
| 22 | ADDISON, AL | 161 | 1942056 | 6473519 | 47 | BRADENTON, FL | 446 | 1023971 | 3413236 |
| 23 | LITHIA SPRINGS, GA | 13 | 1931840 | 6439468 | 48 | LAKELAND, FL | 420 | 1018872 | 3396240 |
| 24 | CALHOUN, GA | 59 | 1822670 | 6075566 | 49 | DAYTONA BEACH, FL | 372 | 1002645 | 3342149 |
| 25 | DELRAY BEACH, FL | 566 | 1795750 | 5985832 | 50 | INDIAN TRAIL, NC | 232 | 967831 | 3226105 |
| 51 | ORANGE CITY, FL | 380 | 961717 | 3205723 | 76 | HOLLYWOOD, FL | 591 | 569317 | 1897723 |
| 52 | GAINESVILLE, FL | 307 | 944789 | 3149298 | 77 | RICHFIELD, NC | 265 | 566254 | 1887514 |
| 53 | AUGUSTA, GA | 142 | 894378 | 2981261 | 78 | FORT WALTON BEACH, FL | 264 | 561067 | 1870223 |
| 54 | MONTGOMERY, AL | 145 | 870201 | 2900669 | 79 | LAKE BUENA VISTA, FL | 408 | 559661 | 1865537 |
| 55 | HAMILTON, AL | 206 | 833402 | 2778006 | 80 | MURFREESBORO, TN | 183 | 553380 | 1844600 |
| 56 | WAYCROSS, GA | 213 | 831064 | 2770214 | 81 | PELHAM, AL | 140 | 549238 | 1830793 |
| 57 | MARYVILLE, TN | 139 | 827265 | 2757550 | 82 | WHITE PINE, TN | 173 | 541269 | 1804230 |
| 58 | MELBOURNE, FL | 449 | 768728 | 2562426 | 83 | BARTOW, FL | 432 | 526340 | 1754468 |
| 59 | BOAZ, AL | 104 | 760642 | 2535472 | 84 | WOODSTOCK, GA | 24 | 526334 | 1754448 |
| 60 | MYRTLE BEACH, SC | 318 | 749015 | 2496718 | 85 | TUCKER, GA | 13 | 517093 | 1723644 |
| 61 | PEMBROKE PARK, FL | 593 | 709433 | 2364776 | 86 | HOLLY HILL, FL | 370 | 490697 | 1635657 |
| 62 | CARY, NC | 349 | 705616 | 2352052 | 87 | HERMITAGE, TN | 209 | 490228 | 1634092 |
| 63 | COLUMBUS, GA | 90 | 683790 | 2279300 | 88 | LAUDERDALE LAKES, FL | 581 | 489961 | 1633204 |
| 64 | APEX, NC | 345 | 682257 | 2274192 | 89 | ARDMORE, TN | 163 | 489226 | 1630752 |
| 65 | BEAN STATION, TN | 189 | 672030 | 2240099 | 90 | PLANT CITY, FL | 419 | 488213 | 1627377 |
| 66 | BUFORD, GA | 34 | 671982 | 2239940 | 91 | KENNESAW, GA | 21 | 486649 | 1622165 |
| 67 | GREENVILLE, SC | 139 | 654889 | 2182964 | 92 | GADSDEN, AL | 93 | 482011 | 1606702 |
| 68 | FRANKLIN, TN | 203 | 640964 | 2136545 | 93 | PORT CHARLOTTE, FL | 487 | 476237 | 1587456 |
| 69 | PANAMA CITY, FL | 258 | 634962 | 2116541 | 94 | SEBRING, FL | 467 | 469563 | 1565211 |
| 70 | SAVANNAH, TN | 240 | 629881 | 2099602 | 95 | DENVER, NC | 228 | 467856 | 1559520 |
| 71 | ORANGE PARK, FL | 295 | 622441 | 2074805 | 96 | MATTHEWS, NC | 232 | 450337 | 1501124 |
| 72 | TAYLORS, SC | 144 | 615734 | 2052448 | 97 | MACON, GA | 78 | 449238 | 1497459 |
| 73 | ANDERSONVILLE, TN | 169 | 604646 | 2015488 | 98 | SAVANNAH, GA | 226 | 448706 | 1495687 |
| 74 | OLD HICKORY, TN | 213 | 590241 | 1967470 | 99 | OLDSMAR, FL | 407 | 447207 | 1490689 |
| 75 | LAKE WORTH, FL | 557 | 586770 | 1955901 | 100 | LAKE CITY, FL | 267 | 446812 | 1489373 |

The collector's total cost data are based on personal communications with Ashman (2007). Some extrapolations and assumptions are made to his initial data. The variable cost consists of the equipment and direct labor components. The fixed cost includes the indirect labor and facility components.

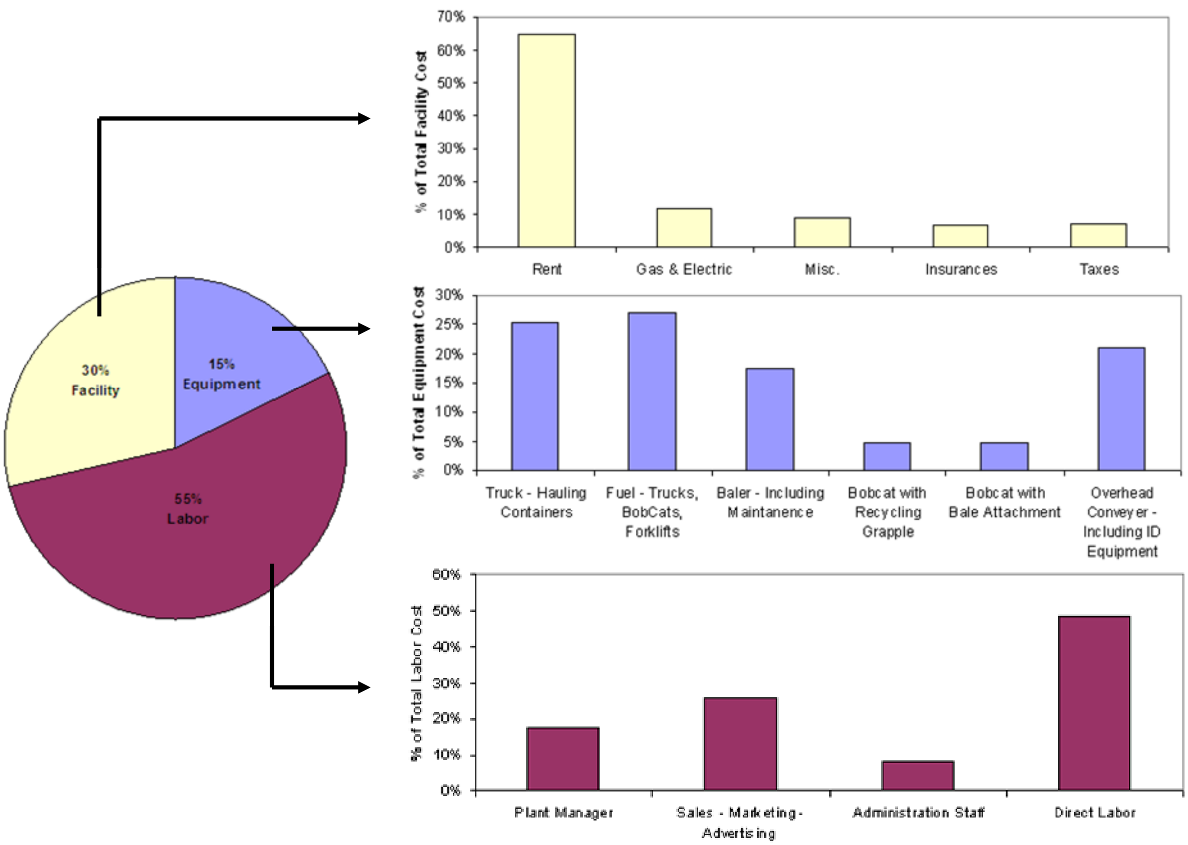



Figure 7.10: Collection Cost Components

The collection equipment cost term includes trucking for hauling containers, fuel, baler and its maintenance, bobcat with recycling grapple, bobcat with bale attachment, and overhead conveyor with identification equipment. All of the above equipment are usually leased. The indirect labor consists of the plant manager, sales persons, and administrative staff. The facility cost contains rent (approximately $\$8/ft^2$), gas, electricity, insurance, taxes and miscellaneous items. Typically equipment, labor, and facility for the northeast region of the U.S. take values of approximately 15%, 55%, and 30% of the total cost, respectively. The breakdown of cost components is presented in Figure 7.10. The collectors' fixed and variable costs, which directly depend on the grouping method, are provided in the following section.

### 7.5.2 Grouping

Given the incomplete nature of the collection problem, it is assumed that the processor has estimations of the fixed and variable costs of each collector group. As a result, the implemented methods in determining groupings are crucial. These grouping methods are subjective in that different processors can entirely have different grouping schemes. Two grouping methods are selected to explore the effects on the case study outcomes.

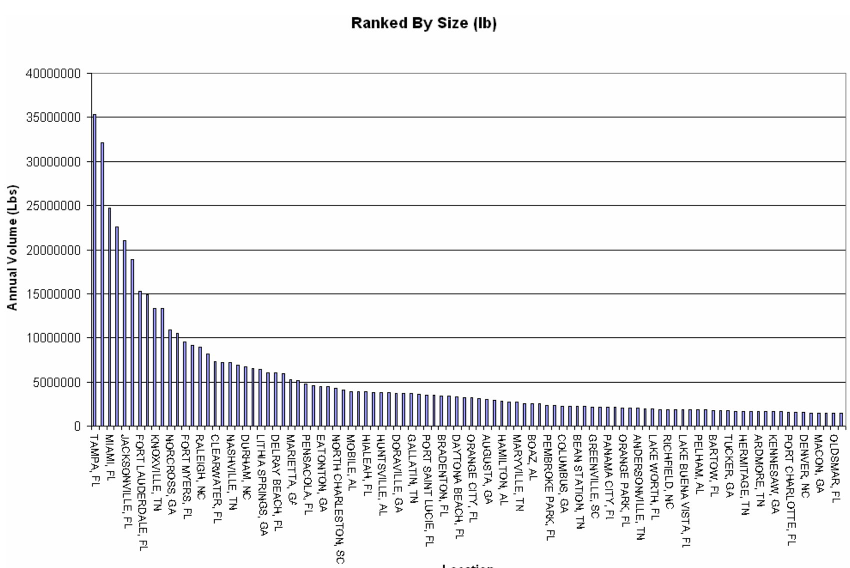


Figure 7.11: Ranked Data by Annual Volume (lb)

The major criterion in choosing a grouping scheme is fairness. Once the grouping is determined, it is intuitive that every collector in the same group should be treated equally. If not, the model may propose different contracts to equivalent collectors. For example, if the region is the criterion, two collection sites in the suburb of Atlanta should receive the same contract. If the one finds out that the other receives a much more attractive contract, he is very likely to reject his contract due to perceived unfairness.

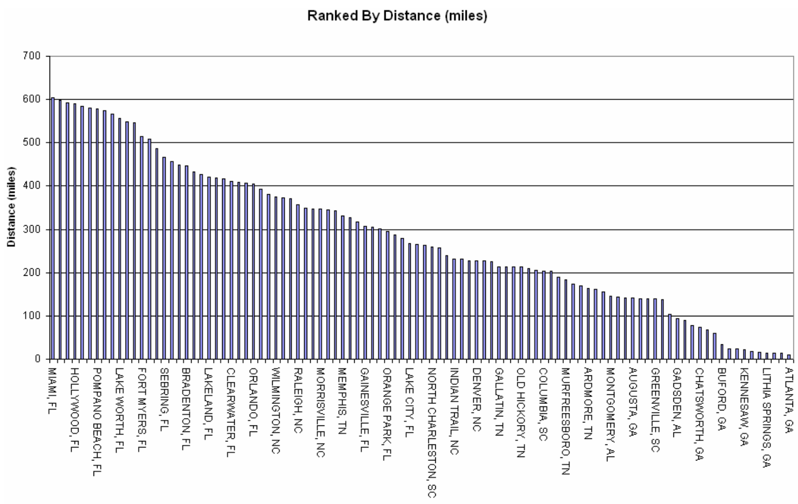


Figure 7.12: Ranked Data by Distance (mile)

The first step in determining grouping criteria for this case study is to analyze the annual volume and distance data, as shown in Figures 7.11 and 7.12. It is apparent that there are a few extremely high volume sites with many low volume sites. As for the distance data, although it is crucial to the model, it does not convey much information for the grouping since state information provides the same information.

Two grouping schemes are proposed. The first grouping method only takes annual volume into consideration, Figure 7.13. The first group, G1, has annual volume exceeding 20M lb/year with 5 sites and has a variable cost (marginal) of \$0.04/lb with a fixed cost of \$495,000. The second group, G2, has annual volume between 10M and 20M lb/year with 7 sites. It has a marginal cost of \$0.05/lb with a \$396250 fixed cost. Finally, the third group, G3, has an annual volume of less than 10M lb/year with 88 sites and marginal and fixed costs of \$0.10/lb and \$237500, respectively. The processor fixed cost for selecting group 1, 2, and 3 are \$1,000,000, \$750,000, and \$1,500,000, respectively.

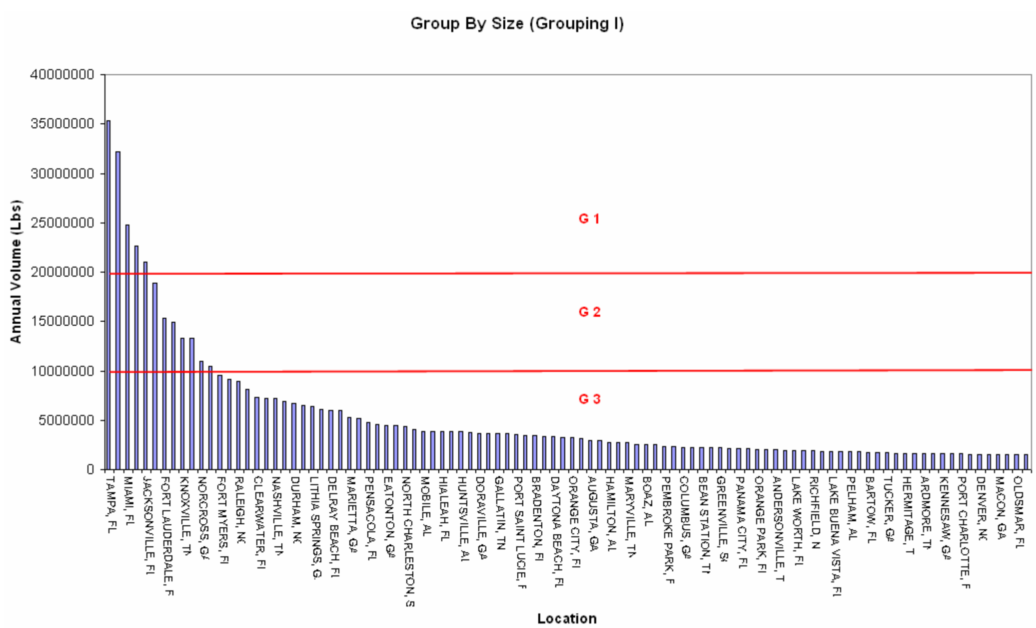


Figure 7.13: First Grouping Scheme

The second grouping system incorporates state information with the annual volume. With various characteristics embedded in the state information; such as population density, transportation infrastructure, and collection policy, it is deemed to be a suitable gauge of similarity. The annual volume is still very important. Hence this second grouping system first distinguishes state and then ranks the annual volume, Figure

7.14. This grouping scheme is summarized in Table 7.2 with description, collector's variable cost, and collector's fixed cost. The processor's fixed cost for opening Group 1 is $1,000,000. For Groups 2, 3, 4, and 5, the processor has a fixed cost of $500,000. Finally, for Groups 6, 7, 8, 9, 10, and 11, the processor's fixed cost is $350,000.

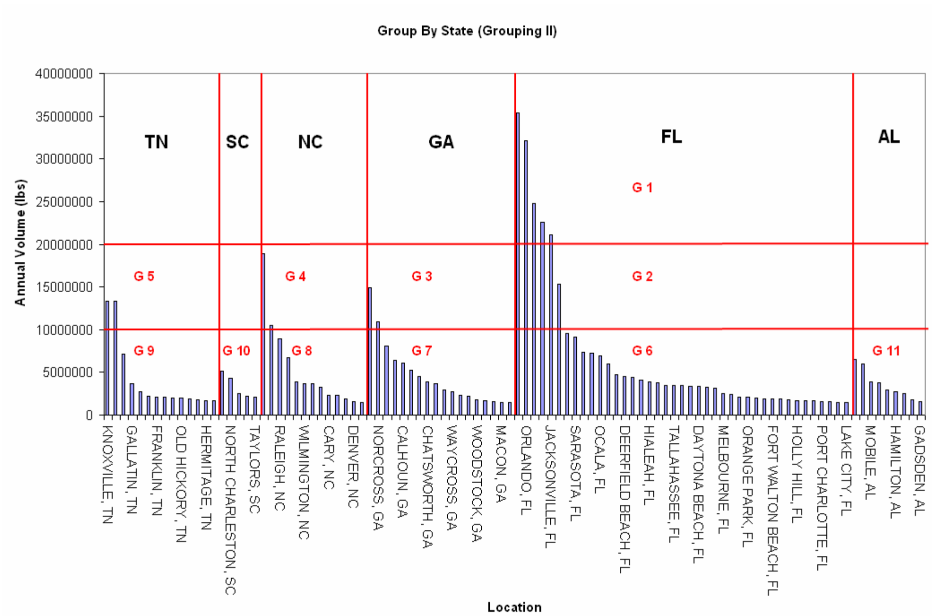



Figure 7.14: Second Grouping Scheme

Although the grouping scheme incorporating state and distance information is also considered, there is a high correlation between state and distance information so that offering one is as good as incorporating both sets of information. Therefore, a distance-based grouping scheme is eliminated. As mentioned earlier, one can easily construct many other grouping schemes depending on what factor is considered critical. However, just these two schemes are analyzed in this case study. With a linear collector cost function, only the fixed and variable costs are needed. From the analysis in Section 7.3, formulations for *NE* quantity and revenue coefficients are calculated. Since Structure 2 has no *NE*s, only two structures are considered. Substituting the input parameters, the *NE* values are summarized in Table 7.3. The material (contract) costs for different groups of Grouping I and II are displayed in Figure 7.15.

Table 7.2: Second Grouping Scheme Summary

| Grouping II | | | | |
|---|---|---|---|---|
| Group | Description | # of Sites | Var Cost | Fixed Cost |
| 1 | Florida, > 20M Annual Volume | 5 | 0.04 | 495000 |
| 2 | Florida, >10M & =< 20M Annual Volume | 1 | 0.05 | 396250 |
| 3 | Georgia, > 10M Annual Volume | 2 | 0.052 | 396250 |
| 4 | North Carolina, > 10M Annual Volume | 2 | 0.054 | 396250 |
| 5 | Tennessee, > 10M Annual Volume | 2 | 0.056 | 396250 |
| 6 | Florida, =< 10M Annual Volume | 35 | 0.1 | 237500 |
| 7 | Georgia, =< 10M Annual Volume | 16 | 0.1 | 237500 |
| 8 | North Carolina, =< 10M Annual Volume | 11 | 0.102 | 237500 |
| 9 | Tennessee, =< 10M Annual Volume | 12 | 0.102 | 237500 |
| 10 | South Carolina | 5 | 0.104 | 237500 |
| 11 | Alabama | 9 | 0.104 | 237500 |

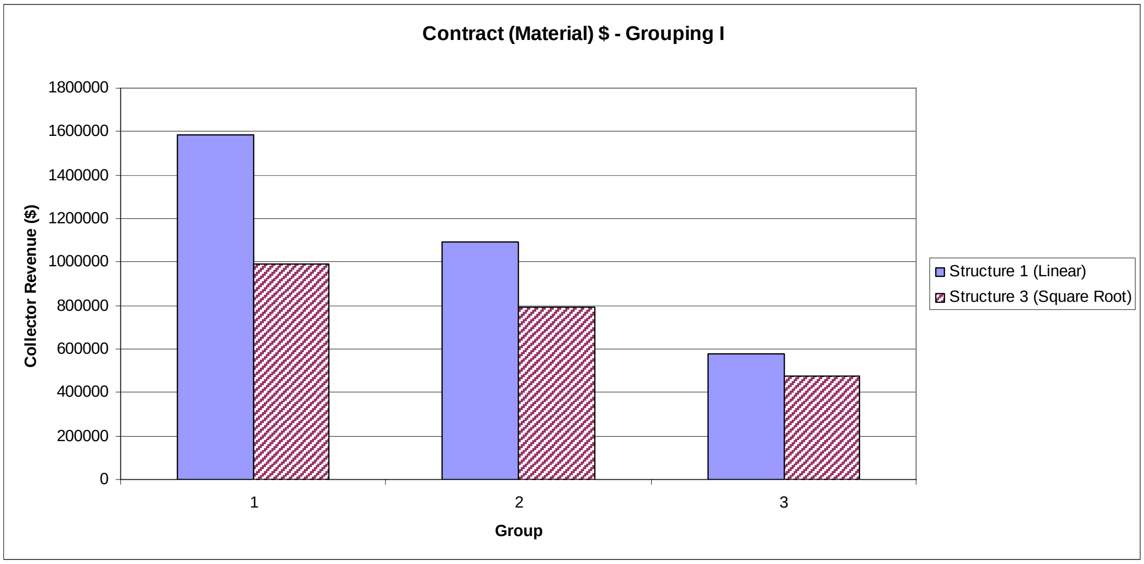
### 7.5.3 Model

In this section, Stage-1 and Stage-2 calculations are discussed. The Stage-1 includes the calculations to find the *NE* quantity and revenue coefficient. For each group, the *NE* values are determined.

Table 7.3: Stage-1 *NE* Values Summary for Grouping I and Grouping II

| Grouping I | Structure1 | | Structure3 | |
|---|---|---|---|---|
| Group | q* | p* | q* | p* |
| 1 | 27178000 | 0.0582 | 12375000 | 281.42 |
| 2 | 13870000 | 0.0786 | 7925000 | 281.51 |
| 3 | 3404000 | 0.1698 | 2375000 | 308.22 |
| **Grouping II** | **Structure1** | | **Structure3** | |
| **Group** | **q*** | **p*** | **q*** | **p*** |
| 1 | 27178000 | 0.0582 | 12375000 | 281.42 |
| 2 | 15286000 | 0.0759 | 7925000 | 281.51 |
| 3 | 12916000 | 0.0827 | 7620192 | 287.09 |
| 4 | 14692000 | 0.0810 | 7337963 | 292.56 |
| 5 | 13294000 | 0.0858 | 7075893 | 297.93 |
| 6 | 3559000 | 0.1667 | 2375000 | 308.22 |
| 7 | 3524000 | 0.1674 | 2375000 | 308.22 |
| 8 | 3599000 | 0.1680 | 2328431 | 311.29 |
| 9 | 2581000 | 0.1940 | 2328431 | 311.29 |
| 10 | 3243000 | 0.1772 | 2283654 | 314.32 |
| 11 | 3536000 | 0.1712 | 2283654 | 314.32 |

These values are entered into the Stage-2 model with parameters from Assavapokee et al. (2007) and Ashman (2007). The Multiple Collector Type Model in Section 7.4.2 is implemented. Stage-2 determines what contract to offer each group and which sites to offer the contract. These results are reported in Section 7.5.4.



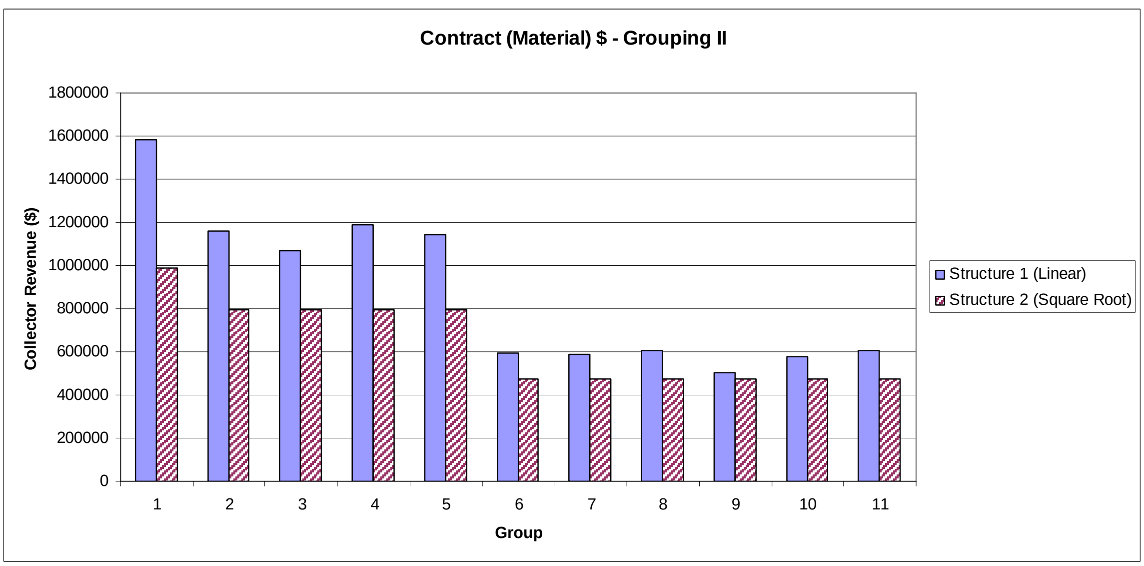




Figure 7.15: Material Contract for Different Groups of Grouping I and II

### 7.5.4 Results

The IP model is solved by GAMS with the CPLEX 9 Solver, CPLEX (2007). The results of the Grouping I, with only annual volume considered, are presented in Section 7.5.4.1. They are followed by the results of the Grouping II with state information and annual volume considered in Section 7.5.4.2.

#### 7.5.4.1 Grouping I - Results

The overall result is that the processor should offer the linear contract (Structure 1) to only Group 1 and the concave contract (square root) to Group 2 and Group 3. Specifically, the processor should offer the contract to the three sites of Tampa, Orlando, and Jacksonville, all in Florida. This solution is presented in Figure 7.16 where the green circle represents processor and blue squares represent selected collector sites.

Figure 7.16: Solution to Grouping I

With all sites in the high annual volume group, each needs to collect approximately 27M lb of Nylon-6 annually. Dealing with a few large collectors in one region has the advantage that the processor need not have a large infrastructure and many administrative employees to control the operations. The breakdown of the processor's revenue and cost components is displayed in Figure 7.17.

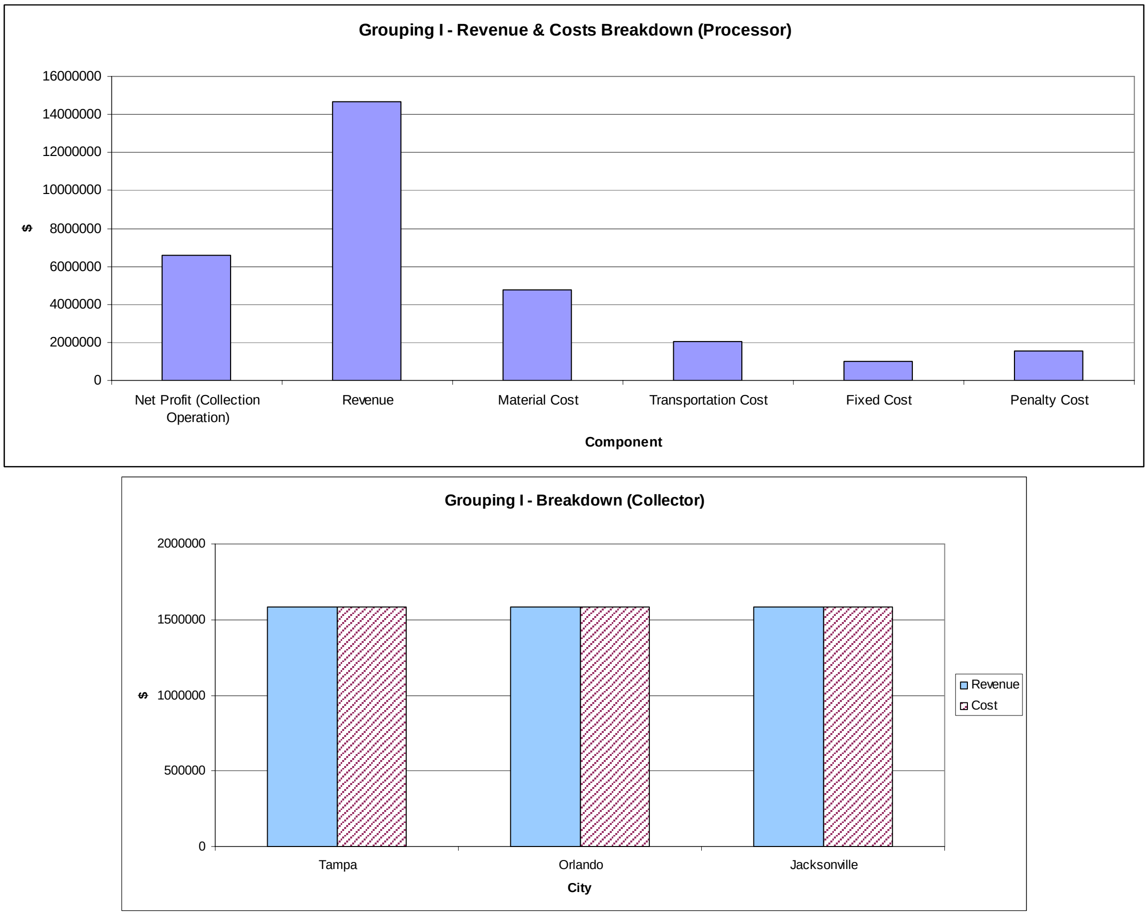



Figure 7.17: Component Breakdown for Grouping I

The material (contract) cost contributes to approximately 50% of the processor's cost and is then followed by the transportation cost, penalty cost for exceeding target quantity, and fixed cost, respectively. The net profit for the collection operation, that is excluding the processing cost, totals to $6,576,000. The selected collectors have a breakeven contract, i.e., they only make the embedded profit from the marginal cost.

To evaluate the solution quality of the Variable Volume Model, a fixed price contract is used as a benchmark. In this fixed price contract, all collector types are offered an identical $/lb contract. With a first-come-first-serve policy, if the collector accepts the contract, all his current material quantity is transported to the processor. A Microsoft Excel simulation is constructed with 10 runs. Each collector quantity is created from a random generator to be within $q_{min}$ and $q_{max}$. Two accepting criteria are implemented. The first criterion considers non-negative collector's net profit, i.e. the collector's fixed cost is considered. With the $0.07/lb target budget, the processor must offer the fixed price of $0.06/lb or less. At $0.06/lb, all 10 runs produce infeasible solutions. The second criterion considers only collector's marginal cost. If a collector has marginal cost greater than $0.06/lb, a contract is rejected. Although omitting collector's fixed cost can lead to negative collector's net profit (eventually defecting from the network), I want to determine the performance of the fixed price contract. Only 3 runs yield feasible solutions ranging in net profit from $5,872,000 to $6,229,000 with an average of $6,062,000. Compared to the net profit of $6,576,000 for the solution obtained using the Variable Volume Model, all these solutions have lower net profits, at an average of 8%. The fixed price contract benchmark shows that the Variable Volume Model can find feasible solutions where the fixed price contract cannot. Also, Variable Volume Model also produces superior solutions (net profit) and gives incentive not to defect from the processor's collection network.

In addition, more experimental runs are performed with different target quantities and budgets. Table 7.4 displays the results of the four additional runs. In Runs 1 and 2, the original budget target of $0.07/lb can be achieved, unlike Run 3 which yields an infeasible solution. Finally, Run 4 has an adjusted budget to be able to find feasible solutions for target quantities of $0.08/lb. Figure 7.18 displays the solutions to Runs 1, 2, and 4.

Table 7.4: Additional Runs to Grouping I

| Additional Runs | | Net Profit (Collection Operation) | Revenue | Material Cost | Transportation Cost | Fixed Cost | Penalty Cost |
|---|---|---|---|---|---|---|---|
| # | Case | | | | | | |
| 1 | Q=100M lb, B=$7M | 5763000 | 18028000 | 6785400 | 2198000 | 3250000 | 153000 |
| 2 | Q=150M lb, B=$10.5M | 12314000 | 27159000 | 10196000 | 2722000 | 1750000 | 884000 |
| 3 | Q=250M lb, B=$17.5M | Infeasible | | | | | |
| 4 | Q=250M lb, B=$20M | 17528000 | 45000000 | 18430000 | 5792000 | 3250000 | 0 |

In Run 1, the target quantity is 100M lb with a budget of $7M. The net profit, revenue, and cost components are shown in Table 7.4. The processor should offer a linear contract to Groups 1 and 2 and a concave contract to Group 3. All three groups are selected. The sites to be offered are Tampa FL, Orlando FL, Jacksonville FL, Knoxville TN, Marietta GA, and Pensacola FL. In Group I, Miami FL and Pompano Beach FL sites are not selected because the processor does not need an additional large collection site to meet the target quantity requirement. Moreover, Miami FL and Pompano FL require a significantly larger transportation cost due to long distances. To avoid large penalty cost for exceeding the target quantity, the Groups 1 and 2 receive concave contracts which require much less quantities. Even though, there are additional fixed costs for selecting Groups 2 and 3, the transportation cost and penalty cost for exceeding target quantity outweigh these fixed costs.

In Run 2, the target quantity is 150M lb with $0.07/lb budget. Groups 1 and 2 receive linear contracts whereas Group 3 gets the concave contract. Only sites from Groups 1 and 2 are selected. The net profit, revenue and cost components are displayed in Table 7.4. The sites from Group 1 include Tampa FL, Orlando FL, and Jacksonville FL. The sites from Group 2 include Charlotte NC, Atlanta GA, Knoxville TN, Norcross GA, and Greensboro NC. In this run, many less efficient sites are chosen (Group 2) to have smaller transportation cost. Moreover, by choosing sites from Group 2, the penalty cost of exceeding target quantity is reduced.

a) Run 1

b) Run 2

c) Run 3

Figure 7.18: Solutions to Additional Runs - Grouping I

In Run 3, the target budget of $0.07/lb cannot be achieved. With this larger target quantity requirement, the processor must have a larger collection infrastructure. Because the cost is not growing linearly, the budget needs to be adjusted. Therefore, the budget is adjusted to be $0.08/lb in Run 4. In this run, only the linear contracts are offered. Sites from all three groups are chosen. All sites from Group 1, including Tampa FL, Orlando FL, Miami FL, Jacksonville FL, and Pompano Beach FL, are selected. Selected sites from Group 2 include Charlotte NC, Fort Lauderdale FL, Atlanta GA, Knoxville TN, Memphis TN, Norcross GA, and Greensboro NC. Finally, five sites from Group 3 with smaller material contract quantity are selected. These are Lithia Springs GA, Marietta GA, Doraville GA, Tucker GA, and Kennesaw GA, which are sites nearby the processor's location. The processor collects exactly 250M lb, hence no penalty cost.

Figure 7.19: Solution to Grouping II

#### 7.5.4.2 Grouping II - Results

In this grouping, the processor offers linear contracts to Groups 1, 3, 5, and 7 and offers concave contracts to Groups 2, 4, 6, 8, 9, 10, and 11. Only sites within Group 1

(Florida-high annual volume) are selected. Moreover, the same three sites with Grouping I results are selected. The selected sites are Tampa FL, Orlando FL, and Jacksonville FL, as shown in Figure 7.19.

The breakdown of the processor's revenue and cost components is displayed in Figure 7.20. The processor and collectors' components are identical to the results in Grouping I because the selected sites belong to the group with the same marginal and fixed costs. If these costs were varied, the results would be different.

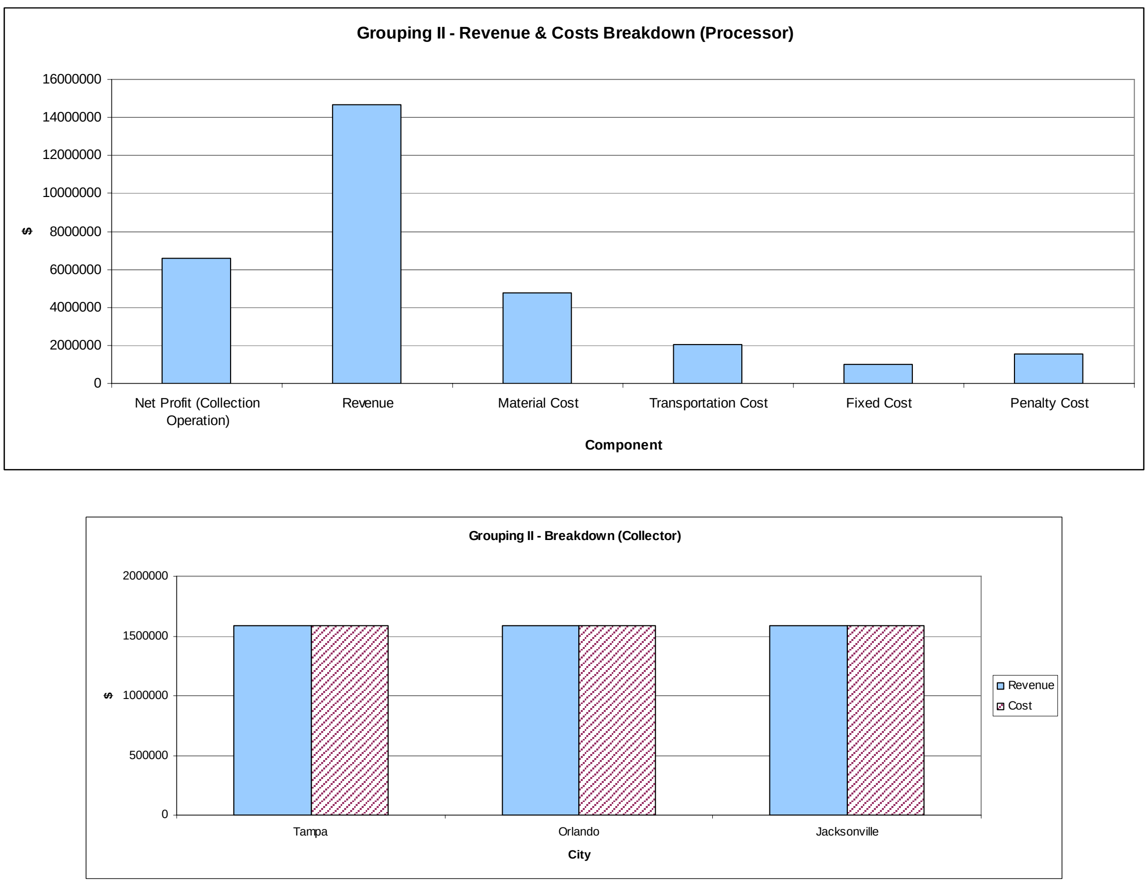



Figure 7.20: Component Breakdown for Grouping II

A fixed price contract ($/lb) is also implemented as a benchmark. Again, a Microsoft Excel simulation model with 10 runs is created. In the first criterion, which considers collector's net profit, all runs yield infeasible solutions. In the second criterion, which only considers marginal cost, only 30% of runs produce solutions. The net profit ranges from $5,672,000 to $5,931,000 with an average of $5,830,000. These solutions have 11% lower net profit than solutions from the Variable Volume Model. It further confirms the superiority of the Variable Volume Model over a simple fixed price contract.

Similar to the previous section, additional runs are conducted with different target quantities and budgets as shown in Table 7.5. In Runs 1 and 2, the original budget target of $0.07/lb can be achieved, unlike Run 3 which yields infeasible solution. Finally, Run 4 has an adjusted budget to be able to find feasible solutions for target quantities of $0.08/lb. Figure 7.21 displays these solutions.

Table 7.5: Additional Runs to Grouping II

| Additional Runs | | Net Profit (Collection Operation) | Revenue | Material Cost | Transportation Cost | Fixed Cost | Penalty Cost |
|---|---|---|---|---|---|---|---|
| # | Case | | | | | | |
| 1 | Q=100M lb, B=$7M | 6925000 | 18275000 | 6606000 | 2438000 | 2000000 | 1526000 |
| 2 | Q=150M lb, B=$10.5M | 10952000 | 27182000 | 9775000 | 4253000 | 2000000 | 1012000 |
| 3 | Q=250M lb, B=$17.5M | Infeasible | | | | | |
| 4 | Q=250M lb, B=$20M | 16962000 | 45108000 | 18816000 | 5859000 | 3350000 | 600000 |

In Run 1, the target quantity is 100M lb with a budget of $7M. The Groups 1, 3, and 7, receive linear contract. The remaining groups are offered concave (square root) contracts. For the network solution, the opened groups include FL-high annual volume, GA-medium annual volume, and TN-medium annual volume. The sites to be offered are Orlando FL, Pompano Beach FL, Jacksonville FL, Norcross GA, and Knoxville TN.

This solution is different than Grouping I in that 1) Pompano FL is selected instead of Tampa FL, 2) Norcross GA (medium annual volume) replaces Marietta GA (low annual volume), and 3) Knoxville TN receives a concave contract. Even though Grouping II solution has 75% higher penalty cost, it yields 30% higher net profit mainly from lower fixed costs. It suggests that grouping methods affect the processor's net profit. Hence, the processor should perform careful analysis in selecting his or her grouping of collectors.

a) Run 1

b) Run 2

c) Run 3

Figure 7.21: Solutions to Additional Runs - Grouping II

In Run 2, the target quantity is 150M lb with a budget of $10.5M. The net profit, revenue and cost components are displayed in Table 7.5. Groups 5, 6, 7, 8, 9, and 10 receive linear material contracts with the remaining groups obtaining concave material contracts. The selected groups are FL-high annual volume, GA-medium annual volume, and North Carolina-medium annual volume. The selected sites are Tampa FL, Orlando FL, Miami FL, Pompano Beach FL, Norcross GA, Charlotte NC, and Greensboro NC. The material contract cost and penalty cost for exceeding target quantity are reduced 11% and 34%, respectively, from Grouping I. However, the transportation cost and fixed cost are increased by 67% and 14%, respectively. As a result, the processor net profit is 7% lower when compared to Grouping I. This suggests that having more groups does not lead to higher net profit.

In Run 3, the target budget of $0.07/lb cannot be achieved. Therefore, the budget is adjusted to be $0.08/lb in Run 4. In this run, linear contracts are offered to all groups, except Groups 6, 8, and 9. All selected sites are offered linear contracts. All sites from Group 1, including Tampa FL, Orlando FL, Miami FL, Jacksonville FL, and Pompano Beach FL, are selected. Selected sites from other groups include Fort Lauderdale FL, Atlanta GA, Norcross GA, Charlotte NC, Greensboro NC, Knoxville TN, Memphis TN, Duluth GA, Lithia Springs, Tucker GA, and Kennesaw GA. Although the penalty cost is significantly higher than Grouping I, the processor net profit is only 2% lower than Grouping I.

## 7.6 Summary and Extensions

Using the Variable Volume Model, the processor can decide which contract structure to offer different collector groups as well as to whom to offer the contract. The concept of fairness is incorporated into the grouping scheme. Depending on processor's knowledge of the collectors' costs, s/he can conduct various grouping methods that will yield different overall results. Thus, the case study is presented as an example of how one can possibly apply the model. Nevertheless, many other contract structures can also be considered. In this case study, the Stage-1 calculations are changed, but with the same solution procedure. The Stage-2 Model then selects the best contract-network decisions that maximize the processor's net profit.

Given the general structure of the Variable Volume Contract Model, it is interesting to examine extensions to a forward supply system. Figure 7.22 depicts a forward supply chain representation, which is a modification of Chopra and Meindl (2004). The forward supply chain stages include supply (components/raw materials), manufacturing, distributing (or wholesale), retailing, and demand. There are transportation links between stages, though some of the stages can be skipped.

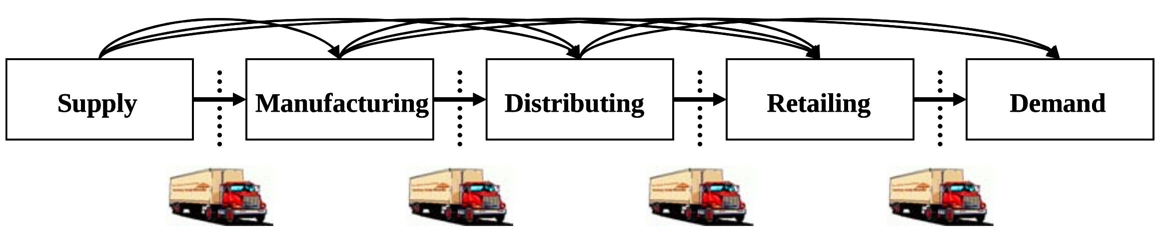



Figure 7.22: Forward Supply Chain Representation

The Variable Volume Model can be extended to particular products in the supply chain, when the products are sold by weight or volume.  Some examples include metal (zinc, copper, and aluminum), jewelry (silver, ruby, and gold), food (meat, dairy, flour, and corn), flowers and plants, and also chemicals.  Because the Variable Volume Contract Model can be applied to pair-wise game, the stage pairings (excluding supply and demand) are analyzed to see applicability.  The limitations to the extension to forward supply system include 1) no competition among collectors, 2) no competition with other processors, and 3) one time period, hence no inventory.

It is demonstrated that all pairings are applicable with the given example(s).  An example of the manufacturer-to-distributor(wholesaler) pair includes many large manufacturers with the ability to choose distributors for the products.  Examples of the wholesaler(distributor)-to-manufacturer pair are oil pipelines with the distributor selecting the manufacturer and also Wal-Mart or Costco selecting the manufacturers. Examples of the manufacturer-to-retailer pair are tire manufacturers choosing Goodyear Tires and Discount Tires but not Pepboys Auto.  Target, Publix, and Kroger are examples of retailers choosing manufacturers for their brand name products.  The wholesaler (distributor)-to-retailer pair example is again an oil pipeline selecting retailers since many retailers are also manufacturers.  Finally, there are numerous instances of large retailers selecting distributors.  In these cases, the Variable Volume Contract and the Strategic Network Model can be applied.  However, the important step is to identify who has the selecting power.  Stackelberg contracts will always follow prestige resources.

This chapter discusses the variable volume contract, which is similar to a typical purchasing quotation in supply management. The Variable Volume Model, which is the major contribution in my thesis, is intuitive and easy to implement. Incorporating the concepts of game theory and Nash Equilibrium, the Variable Volume Model has embedded incentives for collectors to accept the offered contract. Once the contracts are determined, the model assists the processor in determining which collectors to offer contracts in maximizing his or her net profit. The Variable Volume Model is tested on the carpet recycling case study, showing superior performance over a simple fixed price model. Finally, the model can be extended to forward supply chain in a pair-wise game when products are sold by weight or volume.

In Chapter 8, I summarize the different topics in my thesis. The contributions and future directions are discussed.

# CHAPTER 8

# SUMMARY, CONTRIBUTIONS, & FUTURE DIRECTIONS

## 8.1 Summary

This dissertation focuses on the Reverse Production System (RPS) strategic decisions of a processor and collectors when the collection network and contracts for used materials can be co-designed. The three key problems from the processor's point of view are: 1) how to design a strategic collection network; 2) how to be competitive in the collected materials market place when significant investment is at risk; and 3) how to avoid overpaying for materials when collectors are in regions with different volumes and costs. The concepts of mathematical optimization, contract theory, and game theory are utilized in proposing models that couple contract and network problems in lump sum and variable volume contracts. As introduced in Chapter 1, Figure 8.1 displays the research tree.

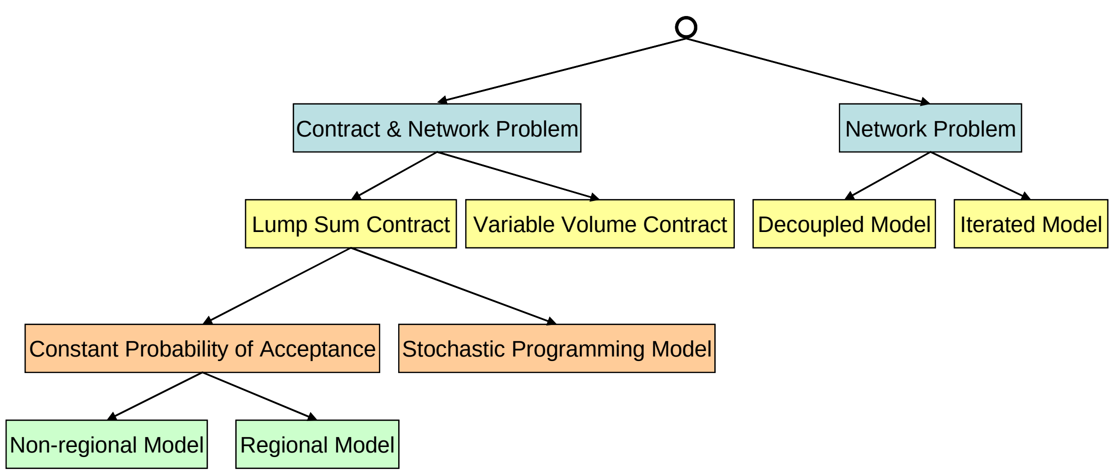



Figure 8.1: Research Tree

In Chapter 1, I provide an overview of RPS problems. The research motivation of the possibility of collectors' defections from the processor's collection network is introduced. The formal research problem with key assumptions and goals is given.

In Chapter 2, I present the research background and a review of the published literature related to various aspects of the research. I begin with the importance of RPS and follow it with a brief literature review of RPS. Since the Variable Volume Contract Model can be generalized to some forward supply chains, the background and literature of Supply Chain Management is also given. The literature review of Mixed Integer Linear Programming Modeling as a decision tool is next presented. With conflicting objectives and asymmetric information as parts of the research problem, contract (incentive) theory is summarized and a literature review of supply chain management contract and principal-agent is given. The dynamic game and Stackelberg game are summarized with discussions of papers of games in reverse logistics. Finally, the Stochastic Programming and Mixed Integer Nonlinear Programming Modeling related to my research are given.

In Chapter 3, the Classical Principal-Agent model, which is the fundamental contract model in my research, is discussed. The model shows how a contract for different collector types can be designed while ensuring participation as well as giving incentives. The Multiple Type Principal-Agent is an extension of the classical model to include more than two collector types. The lump sum contract requires a collector to send a specified quantity to the processor to receive a lump sum monetary transfer. The operation marginal cost plays a major role in defining efficiency. The principal-agent game as well as complete vs. incomplete information concepts are the focus of this chapter. By knowing the net profits of complete and incomplete information, the processor can recognize the maximum cost of information.

In Chapter 4, network problem definitions and models are discussed. The strategic network decisions for RPS include: 1) the number and size of the collection and processing sites; 2) the allocation of processing functions to the sites; 3) the routes for products and materials through potential task network; 4) the modes of transportation used, and 5) the amount of materials allocated to each potential end-use. The Coupled Model was developed earlier by Realff et al. (2000). I introduce the Decoupled Model that initially determines collection decisions which is then followed by processing decisions. I develop an iterative strategy; with clustering techniques for situations where the Coupled and Decoupled Models do not perform well, in the Iterated Model. The very small numbers of specific cases, where the Iterated Model is recommended, are presented. Details of Coupled, Decoupled, and Iterated Models are discussed with numerical studies. I conclude that although the Decoupled Model does not provide optimal solutions, it does provide good solutions with significant execution time reduction in most cases. Finally, the Iterated Model gives the optimal solution with more advantages in computational requirements as the problem size grows.

In Chapter 5, I address the Contract and Network Lump Sum Models. The deterministic mathematical programming approach is presented. Collectors are categorized into types based on their marginal cost. With different space and cost, the ease of collection of each region can vary greatly. Therefore, I present the Regional Principal-Agent Contract in which the processor offers different contracts to different regions. The two strategic network models, Sequential Model and Simultaneous Model, are analyzed. As the name suggests, the Sequential Model solves for contract decisions before inputting into the strategic network model. On the other hand, the Simultaneous Model solves for both contract and network decisions at the same time. I perform numerical studies to find that the Simultaneous Model is preferred because of its ability to more efficiently identify feasible solutions and better (or the same) processor's net profit.

In Chapter 6, the Stochastic Programming (SP) Model for lump sum contracts is presented. This SP Model is different from models in the previous chapters in that there is a solution for each possible scenario (recourse). A two-stage model is implemented in GAMS to test the computational feasibility of the model. In the first stage, the processor determines 1) regions to operate, 2) contracts for each region, and 3) collection hub(s) for each region. The processor then determines the assignment of collections to hubs in the second stage. After the model is described, a numerical study is performed to verify that the SP Lump Sum Model works properly. From these examples, I find that the quantity cannot be trivially set to its upper or lower bounds for efficient and inefficient collectors. The SP Lump Sum Model offers an additional approach for designing a contract and collection network. With uncertainty in each collector's decision to accept the contract, SP can offer complete decisions for all possible scenarios.

In Chapter 7, the Variable Volume Model is studied. Similar to a typical purchasing quotation in supply management, a contract that offers varying prices for different order quantities is analyzed utilizing a two-stage methodology. Stage-1 implements the Stackelberg game which determines Nash Equilibriums (*NE*s) for the collector's quantity and the processor's price. The backward induction procedure is given. With the selected linear collector's total cost function and three selected contract structures, *NE*s are obtained. After studying two different mathematical programming models in Stage-2, I recommend the Multiple Collector Type Model, which is an integer program, be used in this stage. Having decisions from Stage-1 as input parameters, the processor can decide which collectors to offer contracts in maximizing his net profit. A Nylon-6 carpet case study in the southeastern U.S.A. is developed using industry instance data and two different grouping methods. The Variable Volume Model is implemented and tested on the case study. By evaluating with solutions from the baseline simulation where a fixed price ($/lb) is given all collector types, the Variable Volume Model produces feasible solutions which are superior in the processor's net profits. I find that this model can be extended to forward supply chains and next discuss conditions and provide examples.

## 8.2 Contributions

Although several papers employ games in network formation in social networks, markets, and electrical engineering, it is difficult to find contract models in network formation. To my knowledge, this is the first research to incorporate contract and network design in the reverse logistics field. I also do not believe anyone has studied the recycle/reuse collector and processor contract structure as a cooperative model without considering a market.

The major contributions of my research include: 1) integrating the contract and strategic network design in RPS; 2) studying the value of information; 3) developing contract mechanism designs to improve system performance under incomplete information; and 4) introducing and analyzing new strategic RPSs network design models for effectiveness in solution quality and time.

My research investigates the impact of collector type, contract structure, and regional differentiation in different contract models. The lump sum contract structure, using deterministic and stochastic approaches, for processors to develop a large scale network planning model is determined. I further extend this framework to a more typical variable volume contract. As previously mentioned, this model can be extended to forward supply chain contracts under certain assumptions of 1) no competition among collectors, 2) no competition with other processors, and 3) a one-time period decision, hence no inventory levels.

Finally, the construction of a large scale carpet recycling case study shows the usefulness of my research model. Significantly, all models can be extended to many RPS industries once the indices, input parameters, and different decision variables are adapted.

## 8.3 Future Directions

Throughout the chapters of my thesis, the assumption is that only one (recycle/reuse) product is to be collected. It would be quite interesting to model products that consist of many components since there would be more options, such as refurbishing products, sub-modules and components, at the end-of-life of products. With more components, the concepts of correlation and bundle should also be studied. Incorporating these ideas in design contracts is a significantly challenging task to determine collection policy.

I have found that the assumption of a one-time period for the problem of a processor trying to establish a collection network is sensible. However, when the network becomes up and running, it becomes imperative that more time periods be considered. The issues of inventory and additional recruitment are also regarded as important areas in further study. The notion of fairness must be extended to include “inter-generational” issues for collectors since more recently recruited collectors cannot be offered more favorable contracts without the risk of annoying members of the existing network. In addition, the multi-period model must eventually consider the problem of defections from the network.

Collectors are modeled as independent entities. If one considers a company that operates in many regions, then the coordination among collectors must also be considered. This will introduce additional fairness and incentive compatibility constraints to model the fact that different entities could have access to different information if offered different contracts at different scales of operation or in different regions.

Another major future direction is in posing a competition game. With the assumption of collectors independently operating from different regions, competition can be omitted. However, when one considers many collectors wanting to compete for the same recycle/reuse products, a totally different gaming model for the resulting competition game needs to be studied. In such a strategic game, additional indices or additional layers may be required. The idea of market is also definitely needed. With a finite supply (end-of-life products from customers) and a target quantity for the processor of $Q$ (sum of individual quantity) when one collector collects more (less) quantity, other collectors must have less (more) of the quantity pool. This mechanism in the reverse chain needs to be established. With multiple processors, there must be competition among them. At a higher level, multiple processors competing against one another further complicate the model structure.

Another extension is the exploration of uncertainties. Currently, the collector's marginal cost defines the collector types. I believe additional quantitative parameters can help improve identifying collector types. With more accurate type dimensions, different grouping and/or regional settings may provide better recommendations for processors. When there is only partial information on the type, different methodologies must be made. In my models, collector uncertainties include the probability of being efficient and the probability of accepting contracts. However, there are significant uncertainties beyond these. For instance, the market size for the processor has been assumed as fixed and known. This is because the recycled material will often be a small fraction of the overall market and it will fully use whatever its price. There are other situations where the processor will face price-based competition. This will couple the cost of collection to the amount that needs to be collected. These kinds of interaction are beyond the scope of this thesis because the market dynamics are poorly understood.

Another extension is in addressing practical issues in RPS. As many collectors are becoming processors to survive in the used-materials industry, what steps need to be taken? A study of important factors can be very useful to practitioners. Also, cost sharing between a current processor and a collector turning into processor may be considered. Additional to the economic study of the end-of-life products, life cycle assessment (LCA) is important to analyze the environment footprint. LCA is a great tool to help evaluate potential environmental impacts and interpret the results to help make a more informed decision.

In Chapter 5, there are a few models with an MINLP structure. I have not contributed to the MINLP algorithm development, but applied MINLP as the modeling tool for the contract and network problems. However, in solving realistic or large MINLP, algorithm development may be helpful to the processing companies that are trying to establish networks. In Chapter 6, the Stochastic Programming Model requires computation for each scenario. When there are large numbers of scenarios in typical applications, the size of the model becomes rather prohibitive. One proposed approach is the Sample Average Approximation (SAA) algorithm; Kleywegt et al. (2001), Shapiro (1996), Shapiro and Homem-de-Mello (1998), and Wei and Realff (2004).

# APPENDIX A: NOTATION SUMMARY

**Table A.1: Notation Summary by Chapter**

**<u>Contract and Strategic Network Design (Chapter 3, 5, 6, and 7)</u>**

<u>Index</u>

$j$: Index of the region, $j = 1, 2, \ldots, m$

$k$: Index of the distinct set of collector marginal costs, $k = 1, 2, \ldots, m'$, where $m' \leq 2m$.

$i_j$: Index of collectors in region $j$, $i_j = 1, 2, \ldots, I_j$, where $n = \sum_j I_j$

$k_j$: Index of hubs in region $j$, $k_j = 1, 2, \ldots, K_j$

$l$: Index of unique ranking order of marginal costs, $l = 1, 2, \ldots, L$

$s$: Index of scenario, $s = 1, 2, \ldots, S$

<u>Parameters</u>

$ct_j$: Transportation coefficient of region $j$, \$/mile.lb

$c_{truck\ j}$: LTL transportation coefficient of region $j$, \$/mile.lb

$c_j^k$: TL transportation coefficient from hub $k$ to processor of region $j$, \$/mile.tl

$Q$: Target total material quantity required by the processor, lb

$B$: Budget of the processor, \$

$kv$: Coefficient of valuation of processor, \$/lb

$\pi_j$: Probability of a collector in region $j$ accepting the given contract

$p_j$: Probability of a collector in region $j$ being the efficient type

$p_s(s)$: Probability associated with occurrence of scenario $s$

$F_j$: Fixed cost to operate in region $j$, \$

$F_{i,j}$: Fixed cost of collector $i$ of type $j$ to the processor, \$

$F_j^k$: Fixed cost to open hub $k$ in region $j$, \$

$\underline{\theta}_j$: Marginal cost of efficient collector in region *j*

$\overline{\theta}_j$: Marginal cost of inefficient collector in region *j*

$\theta_l$: Marginal cost with the rank order $l^{th}$

$q_{\min j}$: Collector minimum operational quantity of type *j*, lb

$q_{\max j}$: Collector maximum operational quantity of type *j*, lb

$d_i$: Distance between processor and collector *i*, mile

$d_{i,j}$: Distance between collector *i* of type *j* and the processor, mile

$d_j^k$: Distance from hub *k* to processor of region *j*, mile

$d_j^{i,k}$: Distance from collection site *i* to hub *k* of region *j*, mile

$a_j$: Collection cost function coefficient for type *j*, $/lb$^2$

$b_j$: Collection cost function coefficient for type *j*, $/lb

$c_j$: Collection cost function constant for type *j*, $

$I_j$: Total number of collectors of type *j*

$lp$: Lump sum penalty coefficient, $/lb$^2$

$\Delta$: Profit to be shared among collectors, $

*SE*: Shared excess profit fraction to collectors, $0 \le SE \le 1$

<u>Decision Variables</u>

$\underline{t}_j$: Lump sum transfer payment to efficient collector in region *j*, $

$\overline{t}_j$: Lump sum transfer payment to inefficient collector in region *j*, $

$t_l$: Lump sum transfer payment to the rank order *l* collector, $

$\underline{q}_j$: Quantity material collected from efficient collector in region *j*, lb

$\overline{q}_j$: Quantity material collected from inefficient collector in region *j*, lb

$q_l$: Quantity material collected from the rank order *l* collector, lb

$f_i$: Fraction of the profit to be shared to collector *i*

$\pi_c$ : Net profit of the collector, \$

$\pi_p$ : Net profit of the processor, \$

$p_{s,j}$ : Contract revenue coefficient for structure *s* and type *j*

$q_{s,j}$ : Contract quantity for structure *s* and type *j*, lb

$y_j$ : Binary variable of region *j*

$$y_j = \begin{cases} 1 & \text{if contract is offered to region } j \\ 0 & \text{otherwise} \end{cases}$$

$y_{i_j}$ : Binary variable of collector $i_j$

$$y_{i_j} = \begin{cases} 1 & \text{if contract is offered to collector } i \text{ in region } j \\ 0 & \text{otherwise} \end{cases}$$

$y_{s,i,j}$ : Contract offering binary variable

$$y_{s,i,j} = \left\{ \begin{matrix} 1 & \text{if contract structure } s \text{ is offered to collector } i \text{ of type } j \\ 0 & \text{otherwise} \end{matrix} \right\}$$

$z_{s,j}$ : Structure-Type binary variable

$$z_{s,j} = \left\{ \begin{matrix} 1 & \text{if structure } s \text{ is chosen for collector type } j \\ 0 & \text{otherwise} \end{matrix} \right\}$$

**Strategic Network Design (Chapter 4)**

Parameters

$P_{ijt}$ : Price of selling type *j* material at site *i* at time *t*;

$K_i^{(r)}$ : Unit cost per time period for storage at site *i*;

$K_i^{(c)}$ : Unit cost per time period for collection at site *i*;

$K_{ip}$ : Unit flow cost per time period for process *p* at site *i*;

$K_{ii'm}^{(h)}$ : Unit transportation cost per distance from sites *i* to *i'* of transportation mode *m*;

$b_{ii'm}$ : Distance from sites *i* to *i'* by transportation mode *m*;

$\rho_{jp}$ : Proportion of type *j* material consumed by process *p*;

$\rho'_{jp}$ : Proportion of type *j* material produced by process *p*;

$f_i^{(s)}$ : Fixed opening cost of site *i*;

$f_i^{(r)}$ : Fixed storage cost of site *i*;

$f_i^{(c)}$ : Fixed material collecting cost of site *i*;

$f_{ip}$ : Fixed cost of process *p* at site *i*;

$f_{ii'm}^{(h)}$ : Fixed cost of transportation from sites *i* to *i'* by transportation mode *m*;

$\varepsilon_{ijt}^{(c)}$ : Maximum capacity for collection of material type *j* at site *i* at time period *t*;

$\varepsilon_{ij}^{(d)}$ : Maximum amount of material type *j* that can be sold at site *i* at any time period;

$\varepsilon_{ij}^{(r)}$ : Maximum amount of material type *j* that can be stored at site *i* at any time;

$\varepsilon_{ii'm}^{(h)}$ : Maximum amount of material that can be shipped from sites *i* to *i'* by transportation mode *m*;

$\varepsilon_{ipt}$ : Maximum amount of material that process *p* can produce at site *i* at time *t*;

$\alpha_i^{(r)}$ : 1 if storage is allowed at site *i*, and 0 otherwise;

$\alpha_{ij}^{(d)}$ : 1 if material j can be sold at site *i*, and 0 otherwise;

$\alpha_{ii'm}^{(h)}$ : 1 if shipment is allowed between site *i* and site *i'* of transportation mode *m*, and 0 otherwise;

$\alpha_{ip}$ : 1 if process *p* is allowed at site *i*, and 0 otherwise;

$\alpha_{i}^{(c)}$ : 1 if collection is allowed at site *i*, and 0 otherwise;

<u>Continuous Decision Variables</u>

$M_{ijt}$ : The amount of type *j* material collected at site *i* at time *t*;

$S_{ijt}$ : The amount of type *j* material stored at site *i* at time *t*;

$H_{iji'mt}$ : The amount of type *j* material shipped from site *i* to *i'* using transportation mode *m* at time period *t*;

$D_{ijt}$ : The amount of type *j* material sold at site *i* during time period *t*; and

$\xi_{ipt}$ : The extent of process *p* performed at site *i* in period *t*.

<u>Binary Decision Variables</u>

$y_{i}^{(c)}$ : 1 if collection is to be performed at site *i* and equals 0 otherwise;

$y_{ii'm}^{(h)}$ : 1 if shipment is allowed between site *i* and site *i'* of transportation mode *m* and equals 0 otherwise;

$y_{ipq}$ : 1 if replica *q* of process *p* is to be allowed at site *i* and equals 0 otherwise;

$y_{ij}^{(r)}$ : 1 if storage of material *j* is to be allowed at site *i* and equals 0 otherwise;

$y_{ij}^{(d)}$ : 1 if material *j* is allowed to be sold at site *i* and equals 0 otherwise; and

$y_{i}^{(s)}$ : 1 if site *i* is to be opened and equals 0 otherwise.

# APPENDIX B: NEs FOR POLYNOMIAL COST FUNCTION

**Table B.1: Nash Equilibriums for Polynomial Total Cost Function**

Marginal cost function: $C(q) = aq^2 + bq + c$, when $q \geq 0$

$a > 0, b < 0$, and $c > 0$

Total cost function: $TC(q) = aq^3 + bq^2 + cq$

**Structure 1:** Revenue = $pq$, where $p$: \$/lb

<u>Solution 1</u>

$$q_1^* = \frac{1}{9}\,\frac{3b + \sqrt{5b^2 - 9ac + 9ka - 2b\sqrt{4b^2 + 9ka - 9ac}}}{a}$$

$$p_1^* = \frac{1}{9}\,\frac{-2b^2 + 6ac + 3ka - 2b\left(-\frac{1}{3}b + \frac{1}{3}\sqrt{4b^2 + 9ka - 9ac}\right)}{a}$$

<u>Solution 2</u>

$$q_2^* = \frac{1}{9}\,\frac{3b + \sqrt{5b^2 - 9ac + 9ka + 2b\sqrt{4b^2 + 9ka - 9ac}}}{a}$$

$$p_2^* = \frac{1}{9}\,\frac{-2b^2 + 6ac + 3ka - 2b\left(-\frac{1}{3}b - \frac{1}{3}\sqrt{4b^2 + 9ka - 9ac}\right)}{a}$$

<u>Solution 3</u>

$$q_3^* = \frac{1}{9}\,\frac{3b - \sqrt{5b^2 - 9ac + 9ka + 2b\sqrt{4b^2 + 9ka - 9ac}}}{a}$$

$$p_3^* = \frac{1}{9}\,\frac{-2b^2 + 6ac + 3ka + 2b\left(\frac{1}{3}b + \frac{1}{3}\sqrt{4b^2 + 9ka - 9ac}\right)}{a}$$

Solution 4

$$q_4^* = \frac{1}{9}\,\frac{3b - \sqrt{5b^2 - 9ac + 9ka - 2b\sqrt{4b^2 + 9ka - 9ac}}}{a}$$

$$p_4^* = \frac{1}{9}\,\frac{-2b^2 + 6ac + 3ka + 2b\left(\frac{1}{3}b - \frac{1}{3}\sqrt{4b^2 + 9ka - 9ac}\right)}{a}$$

**Structure 2:** Revenue = $pq^2$, where $p$: \$/lb$^2$

Solution 1

$$q_1^* =$$

$$\frac{1}{9}\,\frac{1}{(-c+2k)\,a}\Bigg(-4cb + 2kb + \sqrt{(k+c)^2(18ka + 4b^2 - 9ac)} - \sqrt{\frac{20c^2b^2 - 8cb^2k - 8cb\sqrt{(k+c)^2(18ka + 4b^2 - 9ac)} + 8k^2b^2 + 4kb\sqrt{(k+c)^2(18ka + 4b^2 - 9ac)} - 81k^2ac + 18k^3a - 36ac^3 + 108ac^2k}{(-c+2k)^2}}\;c + 2\sqrt{\frac{20c^2b^2 - 8cb^2k - 8cb\sqrt{(k+c)^2(18ka + 4b^2 - 9ac)} + 8k^2b^2 + 4kb\sqrt{(k+c)^2(18ka + 4b^2 - 9ac)} - 81k^2ac + 18k^3a - 36ac^3 + 108ac^2k}{(-c+2k)^2}}\;k\Bigg)$$

$$p_1^* = -\frac{1}{3}\,\frac{cb + 4kb - \sqrt{(k+c)^2(18ka + 4b^2 - 9ac)}}{-c + 2k}$$

Solution 2

$$q_2^* =$$

$$\frac{1}{9}\,\frac{1}{(-c+2k)\,a}\Bigg(-4cb + 2kb - \sqrt{(k+c)^2(18ka + 4b^2 - 9ac)} - \sqrt{\frac{20c^2b^2 - 8cb^2k + 8cb\sqrt{(k+c)^2(18ka + 4b^2 - 9ac)} + 8k^2b^2 - 4kb\sqrt{(k+c)^2(18ka + 4b^2 - 9ac)} - 81k^2ac + 18k^3a - 36ac^3 + 108ac^2k}{(-c+2k)^2}}\;c + 2\sqrt{\frac{20c^2b^2 - 8cb^2k + 8cb\sqrt{(k+c)^2(18ka + 4b^2 - 9ac)} + 8k^2b^2 - 4kb\sqrt{(k+c)^2(18ka + 4b^2 - 9ac)} - 81k^2ac + 18k^3a - 36ac^3 + 108ac^2k}{(-c+2k)^2}}\;k\Bigg)$$

$$p_2^* = -\frac{1}{3}\,\frac{c\,b + 4\,k\,b + \sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)}}{-c + 2\,k}$$

<u>Solution 3</u>

$q_3^* =$

$$\frac{1}{9}\,\frac{1}{(-c+2\,k)\,a}\left(-4\,c\,b + 2\,k\,b + \sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)} + \sqrt{\frac{20\,c^2\,b^2 - 8\,c\,b^2\,k - 8\,c\,b\sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)} + 8\,k^2\,b^2 + 4\,k\,b\sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)} - 81\,k^2\,a\,c + 18\,k^3\,a - 36\,a\,c^3 + 108\,a\,c^2\,k}{(-c+2\,k)^2}}\,c - 2\sqrt{\frac{20\,c^2\,b^2 - 8\,c\,b^2\,k - 8\,c\,b\sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)} + 8\,k^2\,b^2 + 4\,k\,b\sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)} - 81\,k^2\,a\,c + 18\,k^3\,a - 36\,a\,c^3 + 108\,a\,c^2\,k}{(-c+2\,k)^2}}\,k\right)$$

$$p_3^* = -\frac{1}{3}\,\frac{c\,b + 4\,k\,b - \sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)}}{-c + 2\,k}$$

<u>Solution 4</u>

$q_4^* =$

$$\frac{1}{9}\,\frac{1}{(-c+2\,k)\,a}\left(-4\,c\,b + 2\,k\,b - \sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)} + \sqrt{\frac{20\,c^2\,b^2 - 8\,c\,b^2\,k + 8\,c\,b\sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)} + 8\,k^2\,b^2 - 4\,k\,b\sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)} - 81\,k^2\,a\,c + 18\,k^3\,a - 36\,a\,c^3 + 108\,a\,c^2\,k}{(-c+2\,k)^2}}\,c - 2\sqrt{\frac{20\,c^2\,b^2 - 8\,c\,b^2\,k + 8\,c\,b\sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)} + 8\,k^2\,b^2 - 4\,k\,b\sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)} - 81\,k^2\,a\,c + 18\,k^3\,a - 36\,a\,c^3 + 108\,a\,c^2\,k}{(-c+2\,k)^2}}\,k\right)$$

$$p_4^* = -\frac{1}{3}\,\frac{c\,b + 4\,k\,b + \sqrt{(k+c)^2\,(18\,k\,a + 4\,b^2 - 9\,a\,c)}}{-c + 2\,k}$$

**Structure 3:** Revenue = $p\sqrt{q}$, where $p$: \$/lb$^{1/2}$

$q_1^* =$ No Closed Form Solutions

$p_1^* =$ No Closed Form Solutions

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# VITA

Joshua W. Pas was born in Bangkok, Thailand, on August 30, 1977. With an American father and Thai mother, he spent his early years in Thailand and his post-high school years in America. He received a bachelor's degree in Industrial Engineering from Purdue University, West Lafayette, IN, in 1999, where he enrolled in the honors program and graduated with highest distinction within three years. Because of his long interest in mechanical engineering, he went on to Massachusetts Institute of Technology, Cambridge, MA, to obtain a master's degree in Mechanical Engineering in 2001. At MIT, he concentrated on manufacturing processes, product development, and product design. Eager to gain industry experience, he next joined Bose Corporation, Framingham, MA, as a manufacturing and product development engineer. After two years at Bose, he decided to continue his goal of obtaining a Ph.D. degree in Industrial Engineering. He was delighted to be accepted to Industrial and Systems Engineering at Georgia Tech with a prestigious Presidential Fellowship. There in 2007, he received a master's degree in Industrial and Systems Engineering and a Ph.D. in 2008. His present goal is a job that supports strategic planning, manufacturing, logistics, and/or operations. Apart from his academic interests, he loves watching movies, playing sports (tennis, basketball, and soccer), and walking his basset hound, Maggi.